A numerical study of a vibration-driven mechanism of propulsion in a viscous fluid

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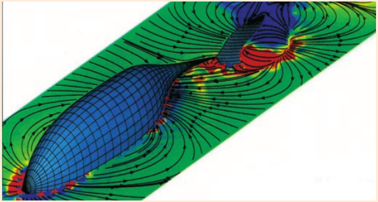
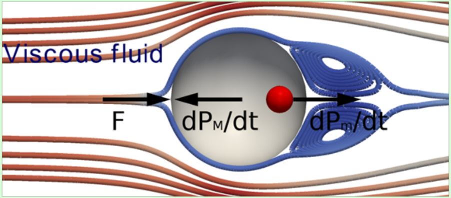
Introduction. Mechanisms of underwater propulsion at low Reynolds numbers $\sim 10^1 - 10^3$

"Reactive motion". Propulsion due to inertial forces



Resestive motion



Introduction. Forces acting on a sphere during the non-stationary translational motion

Asymptotic solution for infinity small Reynolds numbers (Basset 1888)

$$F = C_a \frac{d\tilde{u}}{dt} + C_h \int_{-\infty}^{t} \frac{d\tilde{u}}{d\tau} \frac{d\tau}{\sqrt{t-\tau}} + C_{st}\tilde{u}.$$

Moden model for finite Reynolds numbers

$$F = F_a + F_h + F_{st}.$$

Quasi-stationary resistance F_{st}, added mass force F_a and history force F_h.

Propulsion of a sphere in fluid. Mathematical formulation

Let a spherical body with radius R move in a viscous fluid according to a periodic law $\tilde{u_M}$ with an average (over the period $\tilde{T}$) velocity u_{av} along the axis Ox. Normalizing the spatial coordinates, time, and velocity by R, $R{u_{av}}^{-1}$, u_{av}, respectively, we write the system of equations of motion of the fluid around a spherical body in the following form:

$$\frac{\partial U}{\partial t} + U \cdot \nabla U = -\nabla p + \frac{2}{Re_{av}} \nabla^2 U, \qquad (1)$$

$$\nabla \cdot U = 0.$$

On the boundary of the sphere in the new coordinate system no-slip conditions are specified:

$$u_S = v_S = w_S = 0. \qquad (2)$$

At infinity the change of the velocity is given by the law:

$$u_\infty = -u_M, v_\infty = w_\infty = 0. \qquad (3)$$

The law of a body motion

The acceleration $\dot{u}_M$ of the body is determined by the following continuous piecewise linear function:

$$\dot{u}_M = \frac{\delta u}{c^2} \begin{cases} 0, & \tau \in [0, b] \\ b - \tau, & \tau \in (b, b + c] \\ \tau - b - 2c, & \tau \in (b + c, b + 2c) \\ 0, & \tau \in [b + 2c, 1 - 2c] \\ \tau - 1 + 2c, & \tau \in (1 - 2c, 1 - c] \\ 1 - \tau, & \tau \in (1 - c, 1) \end{cases}, \qquad (4)$$

$$a + b + 4c = 1, \tau = \frac{t}{2\kappa}, \kappa = \frac{\tilde{T} u_{av}}{2R}, \delta u = u_+ - u_-.$$

Function (4) sets the periodic switching between two phases during which the body moves at constant speeds u_+ and u_-.

The condition of steady motion

We assume that the body is in steady motion if the solution of equations (1)-(4) satisfies the conditions:

$$\langle F_x(u_M) \rangle = 0, \langle u_M \rangle = 1. \tag{5}$$

Here F_x is the force acting on the sphere along the axis of oscillation, triangular brackets indicate the averaging over the period.

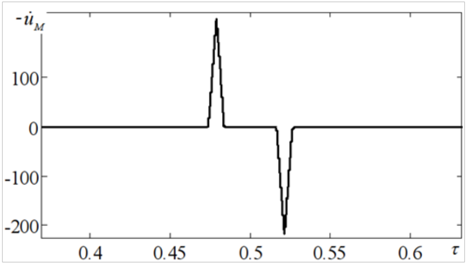


Figure : A fragment of the law of acceleration of the sphere

The calculation of hydrodynamic forces

The calculation of hydrodynamic forces acting on the sphere in the presented dimensionless formulation is carried out according to the formulas:

$$F = (F_x, F_y, F_z) = \frac{2}{\pi}\left(\int_S pnds - \int_S \overline{\sigma} \cdot nds\right),$$

where $\overline{\sigma}$ is a viscous stress tensor, S is the surface of the sphere, $\mathbf{n}$ is the unit normal vector to the sphere surface. It should be noted that the force in the moving coordinate system is determined by the fictitious pressure and therefore contains a contribution from the inertial component. This contribution can be calculated as follows:

$$F_{fk} = \frac{2}{\pi}\dot{u}_M \int_S xnds.$$

As one can see, this term is a linear function of acceleration, and therefore cannot influence on the propulsion.

Numerical scheme

To determine the motion parameters $u_-, \delta u$ that satisfy the conditions (5) for given constants Re_{av}, b, κ the secant method was used. At each iteration of secant method the hydrodynamic problem for determination of F was solved. The scheme was implemented on the basis of a free, open source CFD software OpenFOAM. The standard mesh for three-dimensional calculations contained $3.5 \cdot 10^6$ cells, for axisymmetric case mesh with $4.3 \cdot 10^4$ cells was used.

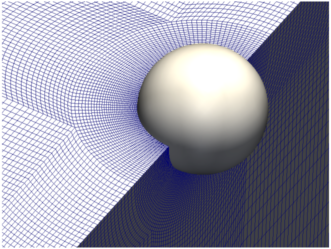
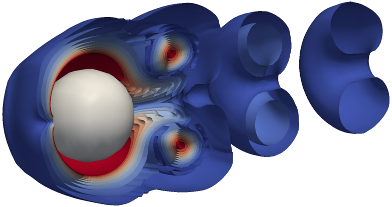


Figure : Three-dimensional mesh structure in the vicinity of the sphere.

Calculations show that in the case when the backward phase is very short ($b \ll 1$), the stability of axisymmetric flow regimes is mainly determined by the Reynolds number Re_+ calculated on the basis of the forward phase speed u_+. We conducted the study in the zone of axisymmetric regimes in the following range of parameters: $15 < Re_{av} < 105$, $0.6 \leq \kappa \leq 6$, $0.04 \leq b \leq 0.2$. On the boundary of this parameter range the stability of the flow to the three-dimensional perturbations was tested using 3D calculations.



Flow structure around the sphere

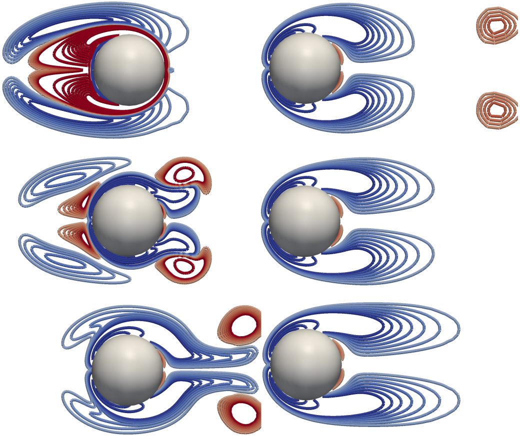


Figure : Instantaneous flow structure. Vorticity. Time moments $\tau = 1/6, 2/6, 3/6, 4/6, 5/6, 1$. $\kappa = 2.2$, $Re_{av} = 47$, $b = 0.08$

Evaluation of hydrodynamic forces. Accelerated motion

Let consider the variations of the forces acting on the sphere during one period in the steady motion. For the convenience of analysis, we divide the entire period into zones corresponding to accelerated motion and motion at the constant speeds.

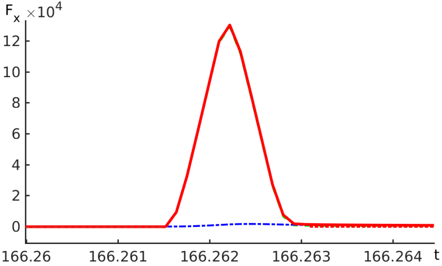


Figure : The structure of hydrodynamic forces in accelerated motion zone. The red solid line indicates the total inline hydrodynamic force F_x, the blue dash-dotted line represents the component associated with the viscous stresses F_v, the green dash line represents the component associated with the pressure distribution F_p. $\kappa = 0.6$, $Re_{\omega} = 25$.

The change of the component $F_p(\approx F_x \approx F_a)$ is completely described by the effect of the added mass. The values of the average coefficient of added mass C_a calculated by the formula (6) at each zone of accelerated motion are very close to theoretical estimate 0.5.

$$C_a = \langle \frac{3}{8} \frac{F_p}{\dot{u}_M} - 1 \rangle \approx 0.5. \quad (6)$$

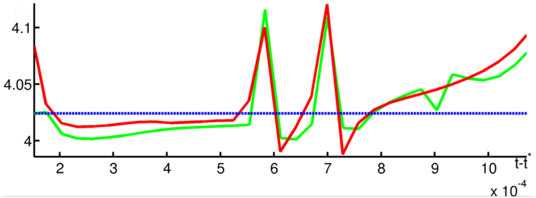


Figure : $\frac{F_p}{\dot{u}_M}$ as a function of time. $\kappa = 0.6$, $Re_{av} = 25$, $b = 0.08$.

The propulsion is the result of forces acting in the forward and backward phases of motion. In both phases, in the absence of acceleration, the force is determined only by the history F_h and quasi-stationary F_{st} components.

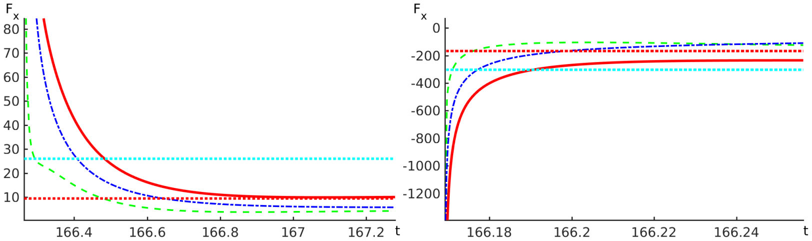


Figure : The structure of hydrodynamic forces in forward and backward motion zones. The red solid line indicates the total inline hydrodynamic force F_x, the cyan doted line represents the average force per phase F_{av}, and the red doted line represents the quasistationary approximation F_{st}. $\kappa = 0.6,\ Re_{av} = 25,\ b = 0.08$.

To calculate the force coefficients on the phases, we use the following formulas:

$$C_d^{+/-} = C_h^{+/-} + C_{st} = \left\langle \frac{F_x}{u_{+/-}|u_{+/-}|} \right\rangle_{+/-} . \qquad (7)$$

Here the triangular brackets with the index $+$ or $-$ indicate the averaging of forces over the corresponding phase. The coefficient of the quasistationary component of the force C_{st} is completely determined by the value of the local Reynolds number in phases. The coefficient of the second component of the force $C_h^{+/-}$ depends on considerably more factors determining the history of movement, including phase duration, Re_+ and Re_-.

Evaluation of hydrodynamic forces. $C_d^{+/-}$ and C_{st}

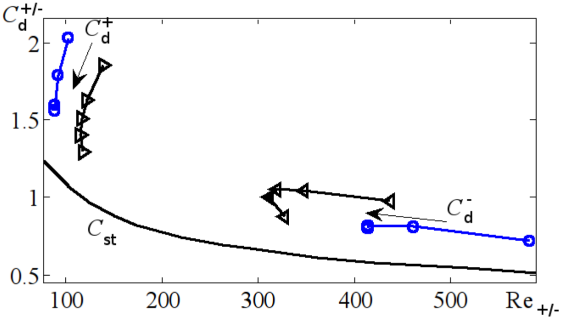


Figure : Values of $C_d^{+/-}$ for $Re_{av} = 47$, $b = 0.16$, $\kappa = [0.7015, 1.403, 2.1045, 2.806, 5.612]$ (black markers) and $Re_{av} = 47$, $b = 0.08$, $\kappa = [1.09, 2.19, 4.38, 5.48]$ (blue markers). The arrows indicate the direction of growth of κ. The black solid line represents the values of C_{st}.

The efficiency of the propulsion mechanism

We introduce the indicator of the efficiency η as the ratio of powers

$$\eta = \frac{N_0}{N_{vbr}}, \tag{8}$$

where N_0 is the minimum power needed to move the body at speed u_{av} and N_{vbr} is the power required by the propulsor to drive the body at this speed under steady-state conditions.

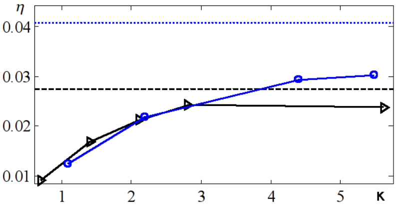


Figure : Efficiency of the propulsion mechanism. $Re_{av} = 47$, $b = 0.16$, $\kappa = [0.7015, 1.403, 2.1045, 2.806, 5.612]$ (black markers) and $Re_{av} = 47$, $b = 0.08$, $\kappa = [1.09, 2.19, 4.38, 5.48]$ (blue markers). The lines represents the estimates of theoretical efficiency.

The efficiency of the propulsion mechanism

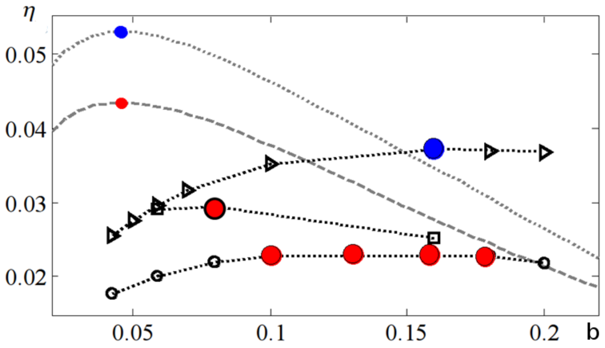


Figure : The efficiency of motion as a function of the phase duration. Black lines with markers represent numerical simulation data: KC = 2.2, Re = 47 (round markers), KC = 2.2, Re = 102 (triangular markers) and KC = 4.4, Re = 47 (square markers).