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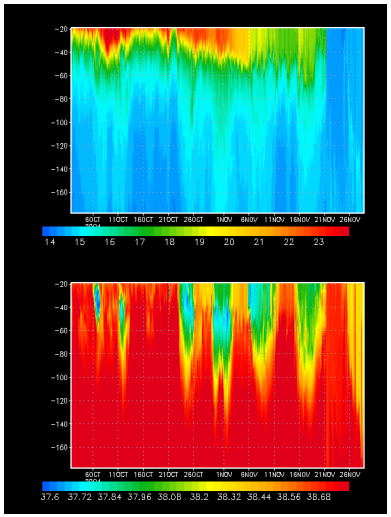
Identification of turbulent model parameters in ocean surface models

Turboradar / Turbident / Urbarisq

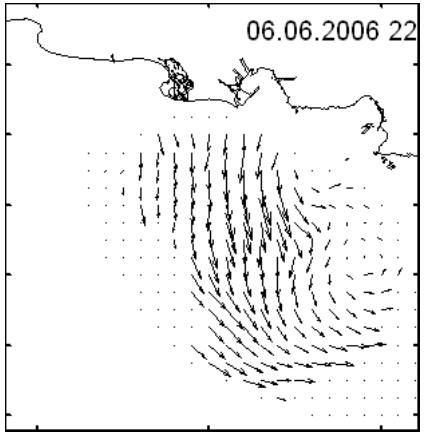
C. Aldebert, M. Baklouti, D. Bourras, H. Branger, T. Caby, J.L. Devenon, D. Faranda, P. Fraunié, R. Fuchs, P. Garreau, G. Koenig, I. Pairaud, V. Rey, A. Sentchev, V. Shrira, S. Vaienti

Surface layer problem



Gliders observations

(Testor et al, 2008)



HF radars observations

(Forget & Shrira, 2015)

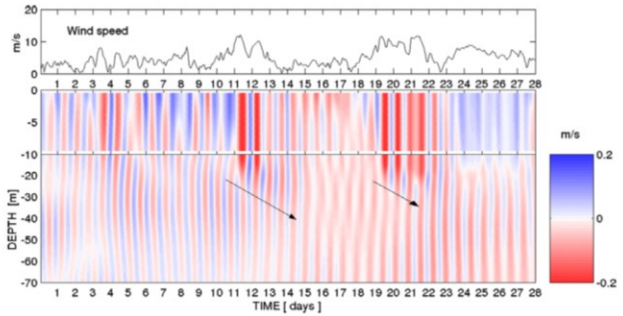
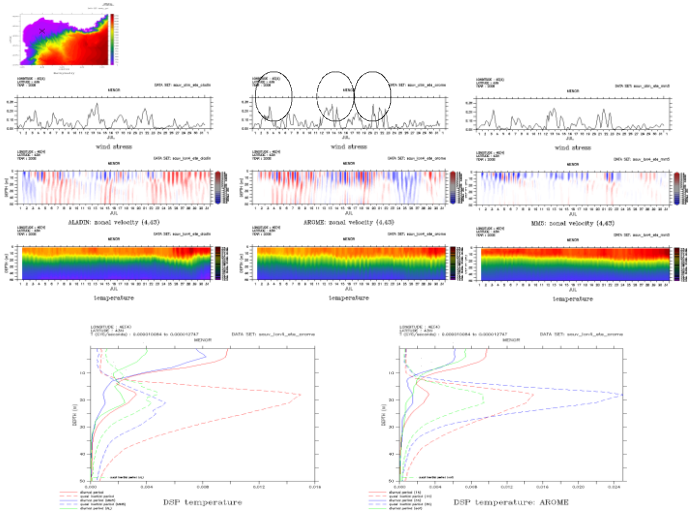


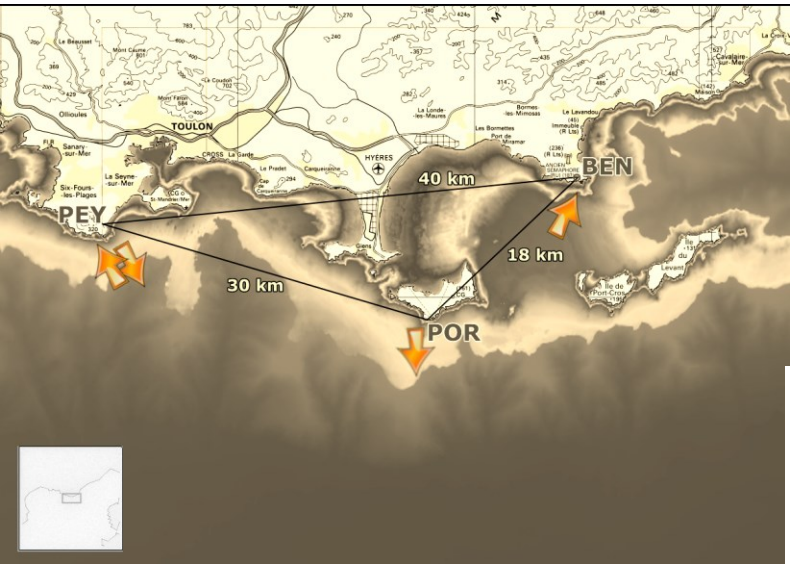
Fig. 19. Time evolution of the wind speed and of the vertical profile of the zonal current at 3.1°E/42.6°N during a 28 day period in summer, in a simulation with the GoL64 ocean circulation model driven with REMO forcing

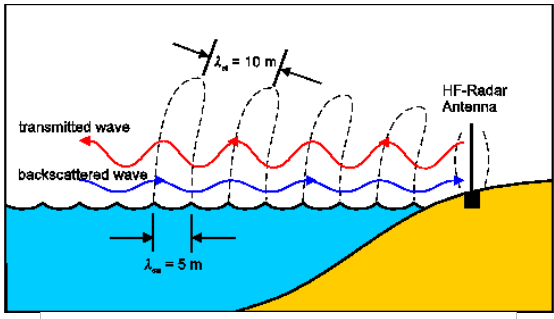
Internal waves permitting models: summer 2008

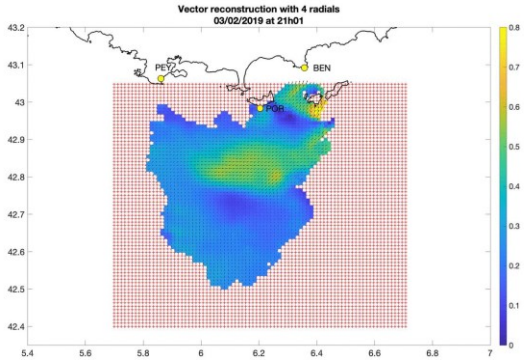
Langlais et al, 2009, Schaeffer et al 2010

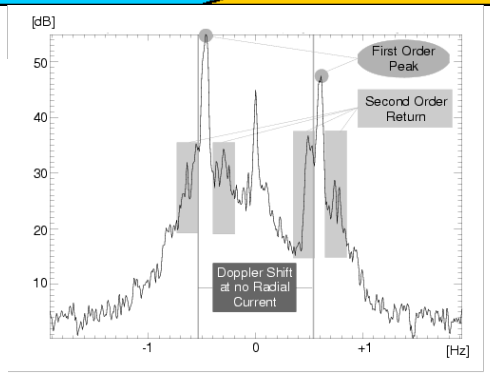


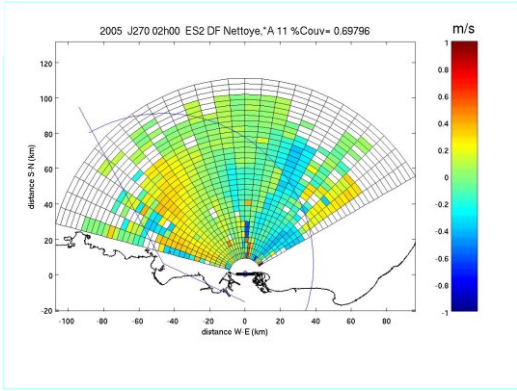
HF radars H current mapping









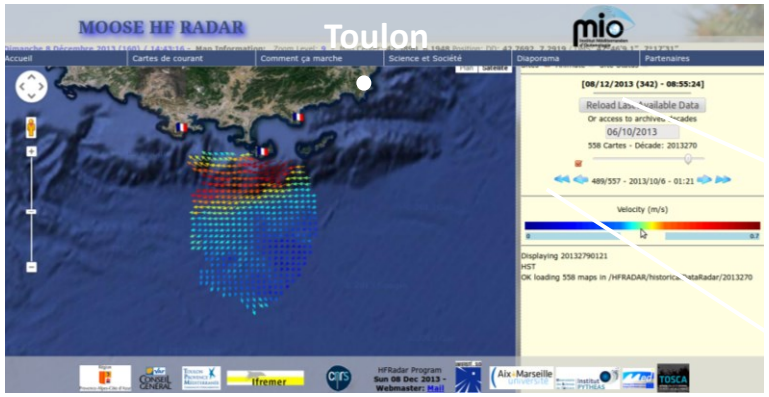


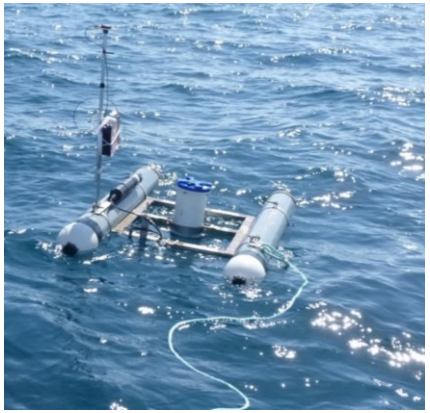
M.I.O. HF Radar team : C. A. Guérin (PR), C. Quentin(IR), A. Gramoullé (IE), D. Dumas(IE), B. Zakardjian(PR), A. Molcard (PR), M. Saillard (PR)

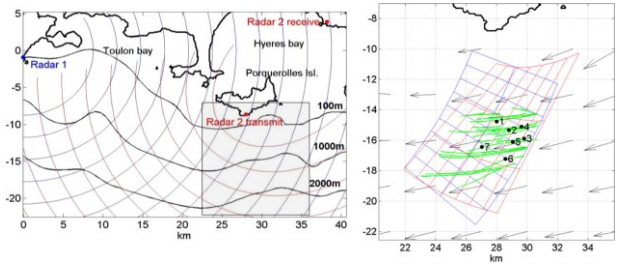
MOOSE / EU MARITTIMO SICOMAR PLUS / ANR TURBIDENT

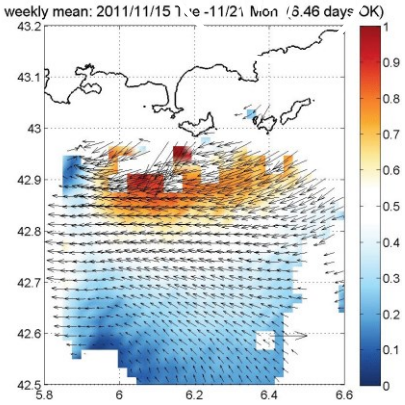
SUBCORAD campaign 2013

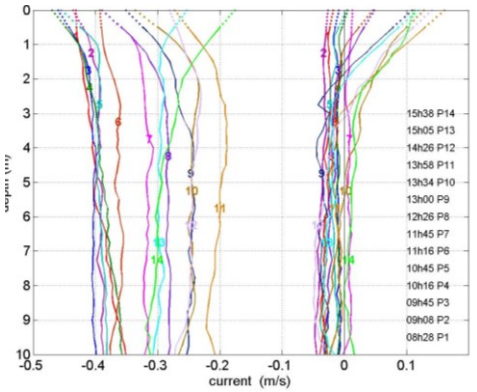
Sentchev, Forget, Fraunié, 2015











The Ekman (1905) solution

$$\frac{\partial u}{\partial t} - fv = \upsilon_t \frac{\partial^2 u}{\partial z^2} \qquad \frac{\partial v}{\partial t} + fu = \upsilon_t \frac{\partial^2 v}{\partial z^2}$$

- **Steady solution**

$$\begin{cases} U(z) = \text{sign}(f)\, U_c \sqrt{2} \exp\left(-\dfrac{z}{d_E}\right)\cos\left(\dfrac{\pi}{4} - \dfrac{z}{d_E}\right) \\ V(z) = U_c \sqrt{2} \exp\left(-\dfrac{z}{d_E}\right)\sin\left(\dfrac{\pi}{4} - \dfrac{z}{d_E}\right) \end{cases}$$

La solution de la spirale d'Ekman est :

$$\begin{cases} u = u_g\left(1 - e^{-\gamma z}\cos\gamma z\right) \\ v = u_g e^{-\gamma z}\sin\gamma z \end{cases}$$

où $\gamma = \left(\dfrac{f}{2K}\right)^{1/2}$

où $K = \nu$

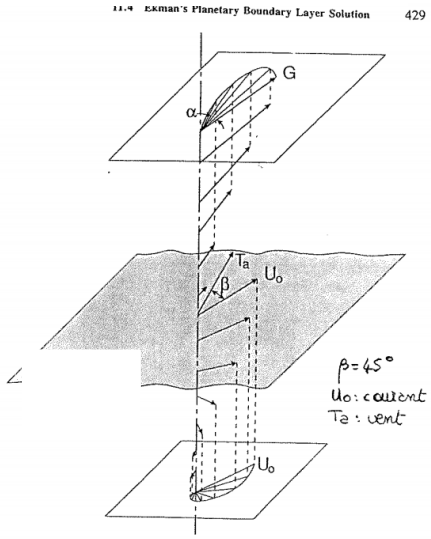
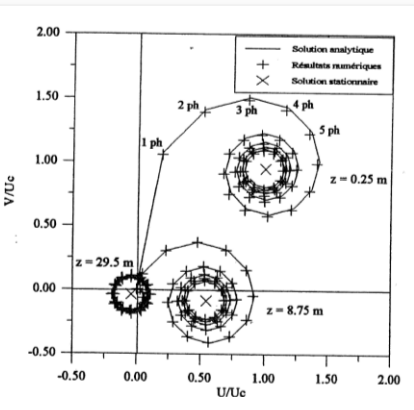


Figure 11.6 Sketch of velocity vectors in the adjacent atmospheric and oceanic PBLs. Note the change in scales.

- **unsteady solution**

$$\begin{cases} U(z,t) = \dfrac{2\,\tau_{sy}}{\rho_r\, d_E\, f}\displaystyle\int_0^{\tilde{t}} \frac{\sin(2\pi\zeta)}{\sqrt{\zeta}} \exp\left(-\frac{z^2}{4\pi d_E^2\,\zeta}\right) d\zeta \\ V(z,t) = \dfrac{2\,\tau_{sy}}{\rho_r\, d_E\, |f|}\displaystyle\int_0^{\tilde{t}} \frac{\cos(2\pi\zeta)}{\sqrt{\zeta}} \exp\left(-\frac{z^2}{4\pi d_E^2\,\zeta}\right) d\zeta \end{cases}$$



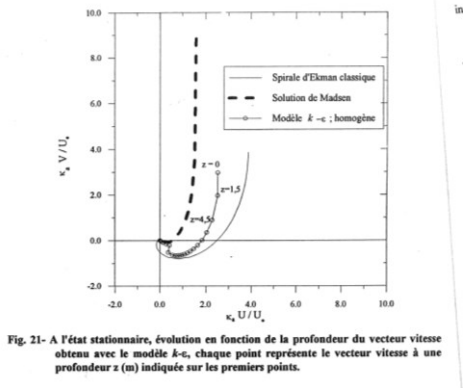
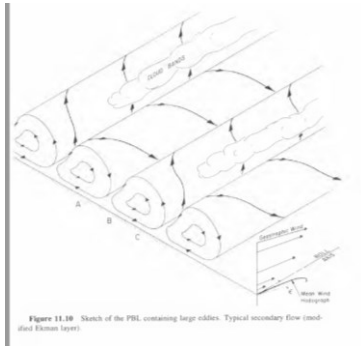
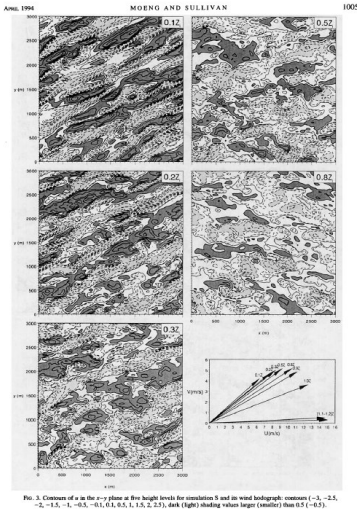


Fig. 21- A l'état stationnaire, évolution en fonction de la profondeur du vecteur vitesse obtenu avec le modèle *k-ε*, chaque point représente le vecteur vitesse à une profondeur z (m) indiquée sur les premiers points.

K-ε model (C. Verdier-Bonnet 1996, 1999)

Ekman layer inflectional Instability





Moeng&Sullivan 1994

- **Ekman Spiral** (Ekman 1905, Gonella 1971, Madsen 1977, Lewis&Belcher 2004, Elipot&Gille 2009, Almelah&Shrira sub)
- **Stability analysis** (Faller 1965, Brown 1974, Leibovich & Lele, 1985)
- **LES** (Deardorff 1970, 1972, Mason & Thomson 1987, Moeng&Sullivan 1994, Zikanov et al, 2003, Sullivan&McWilliams 2010)
- **DNS** (Coleman, Ferziger, Spalart, 1990)
- **Observations PBL** (Le Mone 1973)
- **Observations Ocean** (Price et al 1987, 1999, Csanady 2001)

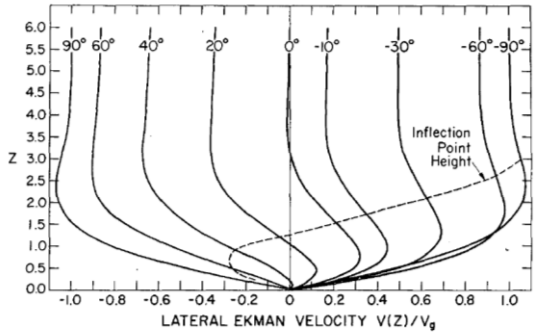


FIG. 1. Two-dimensional velocity profiles taken in vertical planes normal to the roll direction. The angle ϵ denotes the roll angle to the left of the geostrophic velocity. The height and velocity are given in increments of δ and V_g, respectively.

Brown 1972

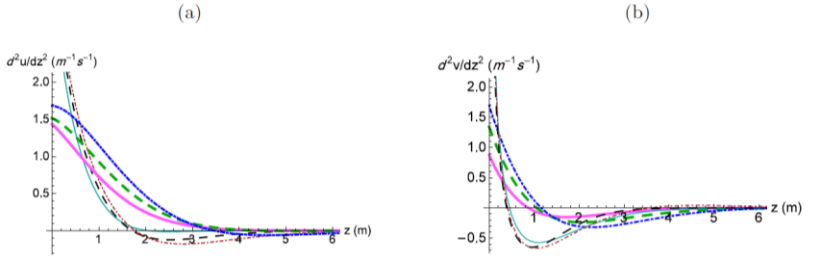
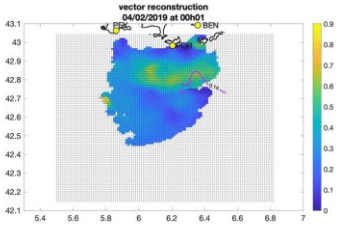
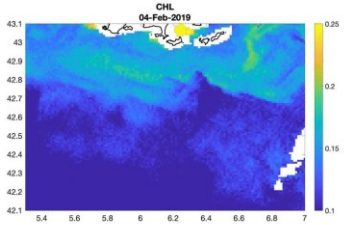


FIGURE 6. Second derivatives of the velocity profiles in two models of viscosity (the classical Ekman model (thick lines) and time-dependent viscosity model (thin lines)) at different times: $\tilde{t} = 2$ (solid lines), $\tilde{t} = 3$ (dashed lines) and $\tilde{t} = 5$ (dotted lines). (a) The second derivative of x-component. (b) The second derivative of y-component. The paramters: $\delta = 3$ hrs, $f = 10^{-4} s^{-1}$, $\nu_0 = 10^{-4} m^2 s^{-1}$.

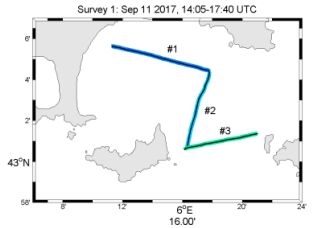
Almelah & Shrira sub

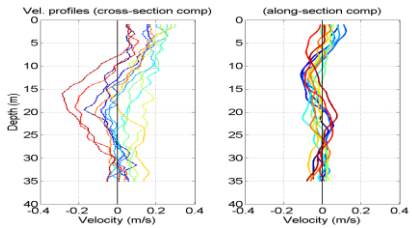
TURBIDENT campaign 2018

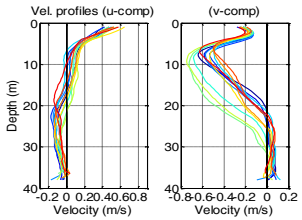


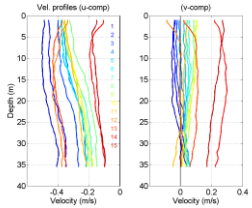


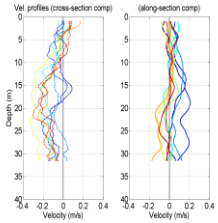
Dumas, Granoullé & Guérin

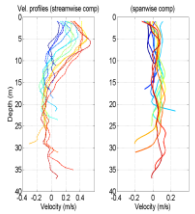


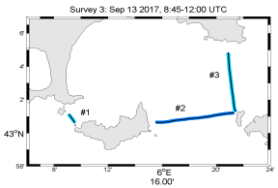












D. Bourras, H. Branger, A. Sentchev

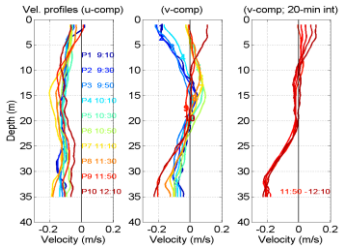
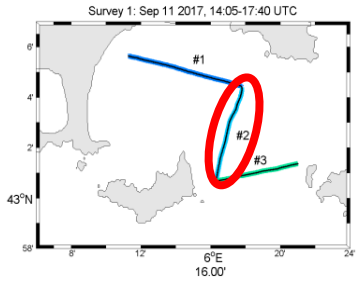


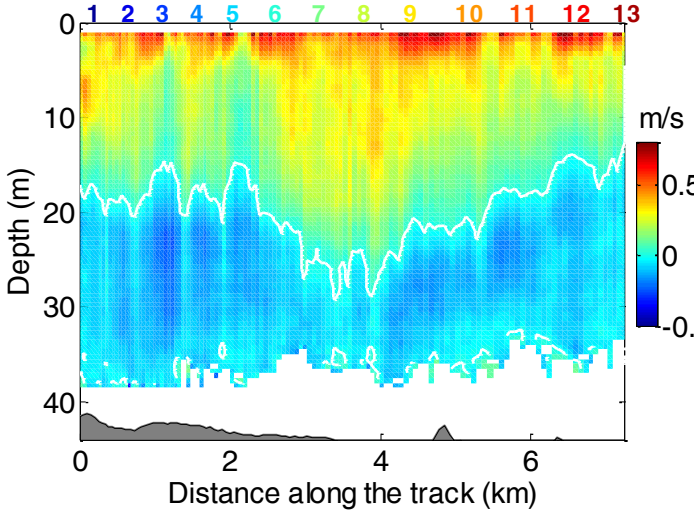
Table 1. Overview of optimization methods, and their pros and cons when applied to complex geophysical models that are costly to program and run. We do not mention Kalman methods, which cannot be used for parameter identification in their current form (Sun et al., 2016).

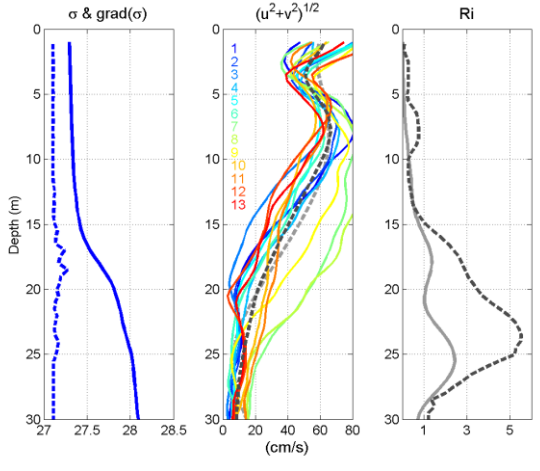
Type	Method	Pros	Cons
By-hand		Easy, only model runs.	Need many runs, unlikely to succeed without a method.
Stochastic, global	Simulated-Annealing	Generic, easy to implement, noisy data.	Need many runs, even for a few parameters.
	Genetic algorithms	Generic, many parameters, noisy data.	Need many runs, hard to tune.
	Hamiltonian Monte-Carlo	Generic, Bayesian framework.	Need many runs
Local gradient descent	Adjoint	Cheap to run, explicit gradient, many parameters.	Costly to build, problem-specific, noise-sensitive.
	Finite-Difference	Generic, easy to implement, almost exact gradient.	Number of runs proportional to the number of parameters, noise-sensitive.
	Simultaneous Perturbation Stochastic Approximation	Generic, easy to implement, cheap to run, many parameters, less noise-sensitive.	Approximative gradient.

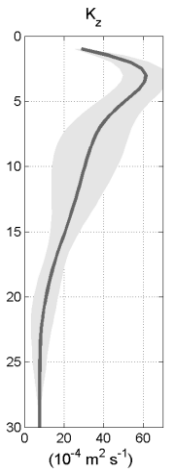
Eddy viscosity estimate



U-comp







From 1 D Ekman model (A Sentchev)

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Parameter estimation of turbulent closure schemes in marine circulation models using Simultaneous Perturbation Stochastic Approximation method : a proof-of-concept.

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(1) Mediterranean Institute of Oceanography, Aix-Marseille University, Toulon University, CNRS/IRD, France, (2) IFREMER Lerpac, La Seyne, France, (3) IFREMER LPOS, Brest, France

The operational problem

- predictions by numerical models of marine circulation strongly rely on the description of the sea-air interface
- difficult to acquire reliable data close to sea surface ⇒ practical challenge is to parameterize the turbulent closure schemes modelling the upper mixing layer
- TURBIDENT project aims to optimize turbulent closure schemes with a new well-suited data set from HF radars and AUV-mounted ADCP
- we use a new data assimilation method that has never been used for turbulent closure schemes optimization

Why this new method ?

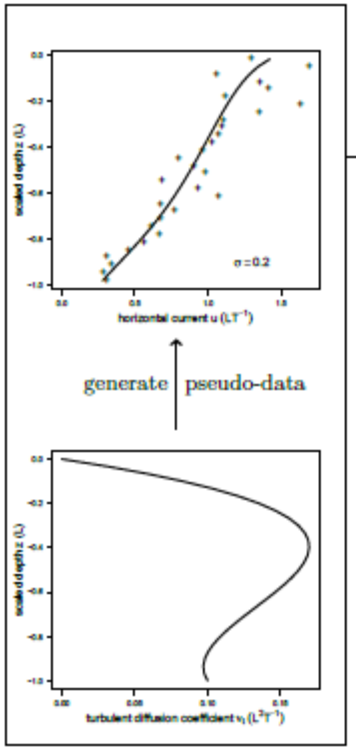
- model derivatives with respect to the parameters to optimize are usually computed explicitly by an adjoint model that need to be developed and maintained
- the SPSA methods (Zhu & Spall, 2002) are based on a small number of model estimates (random parameter perturbations) to approximate at low numerical cost the derivatives by finite differences
- due to the random component, different methods (first and second-order, "modified") has been proposed to improve the procedure (numerical cost, stability)

Preliminary results

- we identified the most stable SPSA algorithm through theoretical examples of the surface layer
- ongoing tests for KPP model (link with interior domain, surface layer thickness)

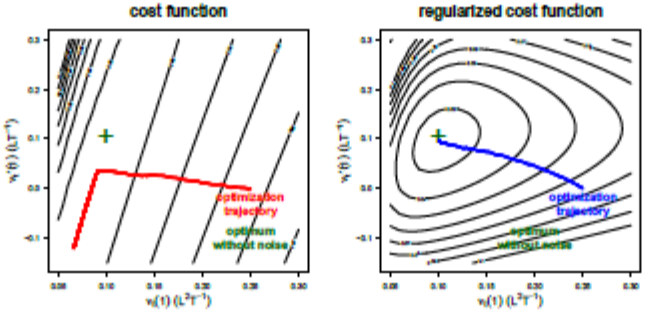
Next steps

- stratified and/or unsteady situations
- sensitivity to sampling under realistic scenarios
- coupling SPSA with existing methods
- same work with k-ϵ model



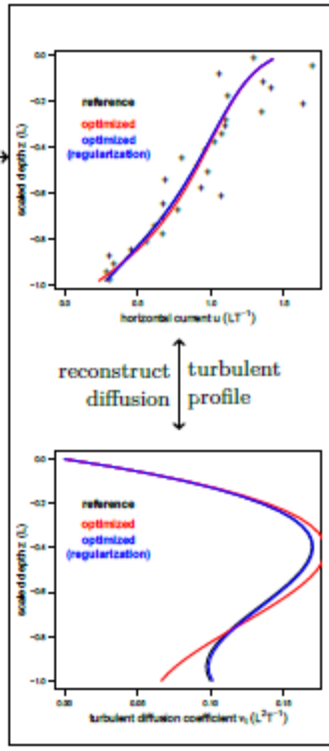
data on current profile

as constraints



optimize parameters

to fit current profile



A steady-state example

Horizontal current (u) experiencing a turbulent diffusion (coefficient ν_t) along a scaled vertical axis ($z \in [0,1]$) :

$$\frac{\partial}{\partial z}\left(\nu_t(z)\frac{\partial u}{\partial z}\right) = 0, \text{ with the boundary condition } u(0) = u_0$$

Like in KPP model (Large *et al.*, 1994), $\nu_t(z)$ is taken as a third-order polynomial that depends on two parameters : its value $\nu_t(1)$ and derivative $\nu'_t(1)$ at the lower boundary (continuity with the interior profile in KPP model)

For reference parameters, we compute the ν_t and u profiles. The current profile is sampled at depths z_i and a random noise is added to generate pseudo-data :
$u_i^{obs} = \alpha_i u(z_i)$, $\alpha_i \sim \mathcal{N}(1, \sigma)$

Outline of SPSA methods

For a set of parameters to optimize θ (here $\theta = (\nu_t(1), \nu'_t(1))$), we define a cost function, here : $S(\theta) = \sum_{i=1}^{n} \left(u_i^{obs} - u_i^{mod}(\theta)\right)^2 / n$.

Starting from initial parameter values, each step involves :
1. simultaneous random perturbations : $\Delta\theta_i = \pm 1$ (equiprobability)
2. use $S(\theta \pm c\Delta\theta)$ to approximate the gradient by finite differences
3. optionnal : similar method for the Hessian (add 2 model runs)
4. use 2. and 3. to compute new parameter estimates (with checking)

For step 4., the modified SPSA method is the most stable :
$\theta^{new} = \theta - a\,\mathrm{grad}(S(\theta))$, where a is the mean of Hessian eigenvalues

One may include additionnal information in the cost function to regularize it, here (as example) :
$S(\theta) = \sum_{i=1}^{n} \left(u_i^{obs} - u_i^{mod}(\theta)\right)^2 / n + \lambda(\nu'_t(1) - \nu'^{obs}_t(1))^2$.

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Classical standard values of TKE turbulent closure model parameters

Table 2. Model parameters.Gaspar et al. (1990)

Parameter	Value	Unit	Meaning
c_k	0.1	-	TKE constant (eddies diffusivity)
c_ε	0.7	-	TKE constant (turbulent dissipation)
$K_{m_{\min}}$	$3\,10^{-5}$	$\mathrm{m}^2.\mathrm{s}^{-1}$	minimal value for moment diffusivity
$\overline{e}_{\min}$	$2\,10^{-6}$	$\mathrm{m}^2.\mathrm{s}^{-2}$	minimal value for TKE
$\overline{e}_{\min 0}$	10^{-4}	$\mathrm{m}^2.\mathrm{s}^{-2}$	minimal value for TKE at surface
Pr_t	1	-	Prandtl number
K_{ratio}	1	-	ratio between TKE and momentum diffusivities
bb	3.75	-	constant to compute surface TKE from wind stress

Simple hydrodynamic model 1DV

- T temperature ; S salinity ; U=(u,v) and w horizontal and vertical velocity , I solar irradiance, F_{sol} solar constant, cp specific heat.

$$\frac{\partial \overline{T}}{\partial t} = \frac{F_{\mathrm{sol}}}{\rho_0 c_p} \frac{\partial I}{\partial z} - \frac{\partial \overline{T'w'}}{\partial z},$$

$$\frac{\partial \overline{S}}{\partial t} = -\frac{\partial \overline{S'w'}}{\partial z},$$

$$\frac{\partial \mathbf{U}}{\partial t} = -f\mathbf{k} \times \mathbf{U} - \frac{\partial \overline{\mathbf{U}'w'}}{\partial z},$$

Turbulent closure scheme

Eddy diffusivities: K_h, K_s, K_m

$$\overline{T'w'} = -K_h \frac{\partial \overline{T}}{\partial z}, \; \overline{S'w'} = -K_s \frac{\partial \overline{S}}{\partial z}, \; \overline{\mathbf{U}'w'} = -K_m \frac{\partial \overline{\mathbf{U}}}{\partial z},$$

$$K_m = \max\left(K_{m_{\min}}, c_k \, l_k \, \sqrt{\overline{e}}\right),$$

$$K_s = K_h = \frac{K_m}{Pr_t},$$

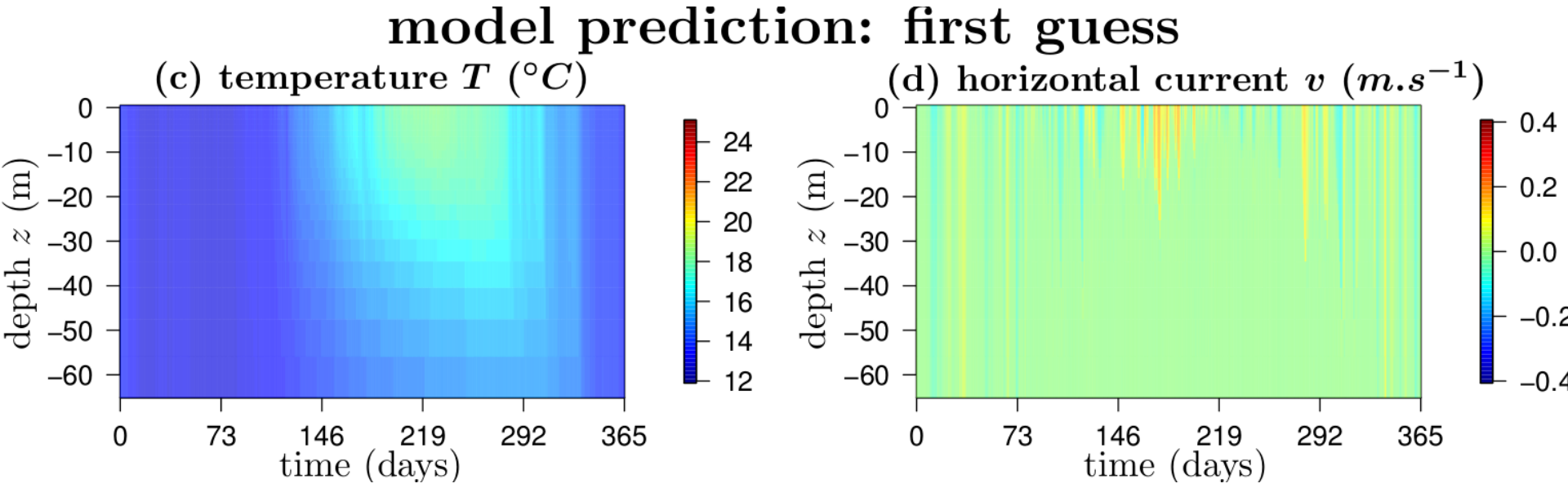
Turbulent closure scheme

$$e = 0.5(u'^2 + v'^2 + w'^2) \qquad \varepsilon = c_\varepsilon \frac{\overline{e}^{3/2}}{l_\varepsilon},$$

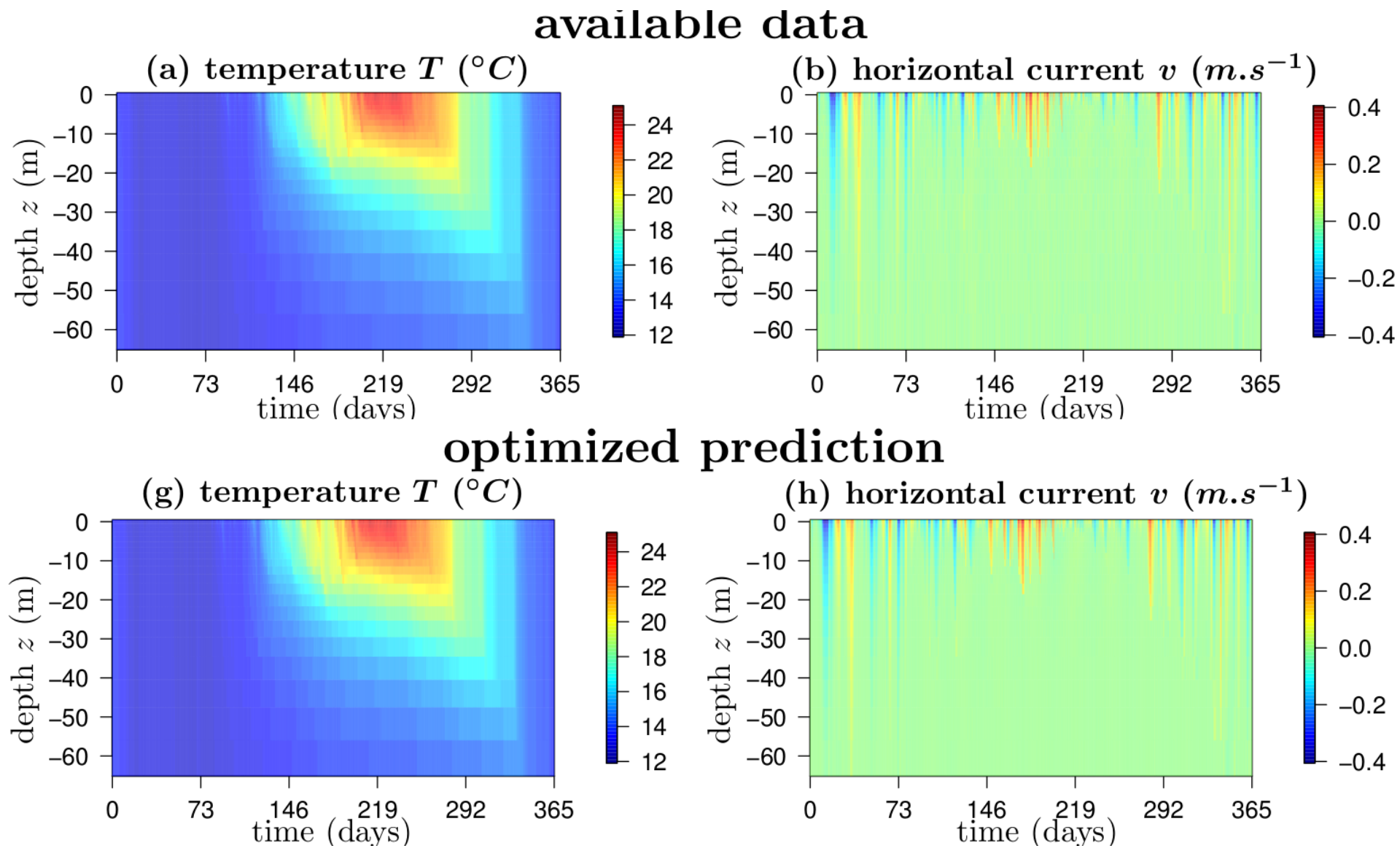
$$\frac{\partial \overline{e}}{\partial t} = \frac{\partial}{\partial z}\left(K_e \frac{\partial \overline{e}}{\partial z}\right) - \overline{\mathbf{U}'w'} \cdot \frac{\partial \mathbf{U}}{\partial z} + \overline{b'w'} - \varepsilon$$

$$\overline{e}_{(z=0)} = \max\left(\overline{e}_{\min 0}, bb\frac{\sqrt{\tau_x^2 + \tau_y^2}}{\rho}\right)$$

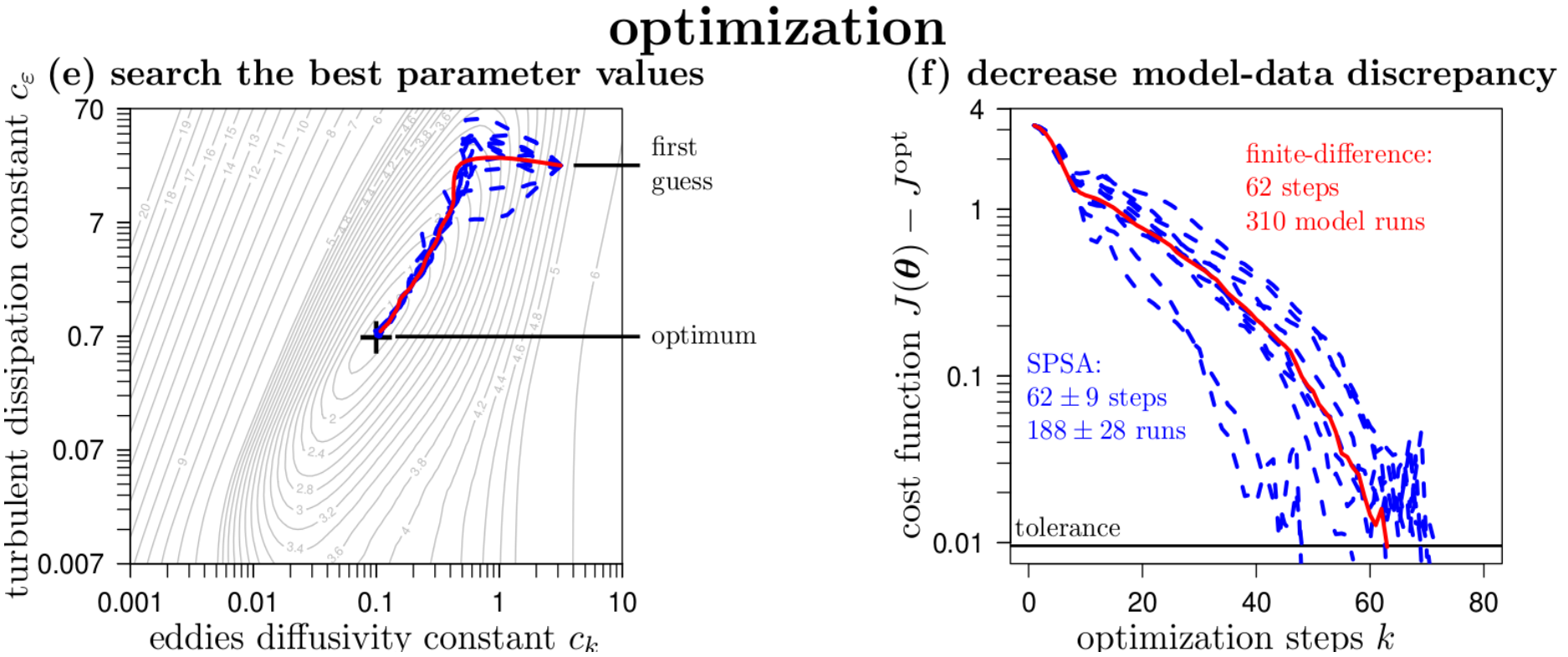
Optimization : arbitrary starting state



Optimization : Twin experiments



Optimal search in the control space example in a 2D sub-space



Cost function and minimization algorithm

(Nesterov Momentum)

$$J(\boldsymbol{\theta}) = \alpha_{uv}\left(\sum_{i=1}^{n}\left(u_i^{\mathrm{pred}}(\boldsymbol{\theta}) - u_i^{\mathrm{obs}}\right)^2 + \sum_{i=1}^{n}\left(v_i^{\mathrm{pred}}(\boldsymbol{\theta}) - v_i^{\mathrm{obs}}\right)^2\right)^{1/2}$$

$$+\alpha_T\left(\sum_{i=1}^{n}\left(T_i^{\mathrm{pred}}(\boldsymbol{\theta}) - T_i^{\mathrm{obs}}\right)^2\right)^{1/2} + \alpha_S\left(\sum_{i=1}^{n}\left(S_i^{\mathrm{pred}}(\boldsymbol{\theta}) - S_i^{\mathrm{obs}}\right)^2\right)^{1/2},$$

procedure starting from an arbitrary initial guess of the parameter values $\boldsymbol{\theta}^{(0)}$:

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - a^{(k)}\boldsymbol{z}^{(k)}. \qquad (2)$$

$a^{(k)}$: step size $z^{(k)}$ searching direction

$$\boldsymbol{z}^{(k)} = \beta\boldsymbol{z}^{(k-1)} + \nabla J(\boldsymbol{\theta}^{(k)} - a^{(k)}\boldsymbol{z}^{(k-1)}).$$

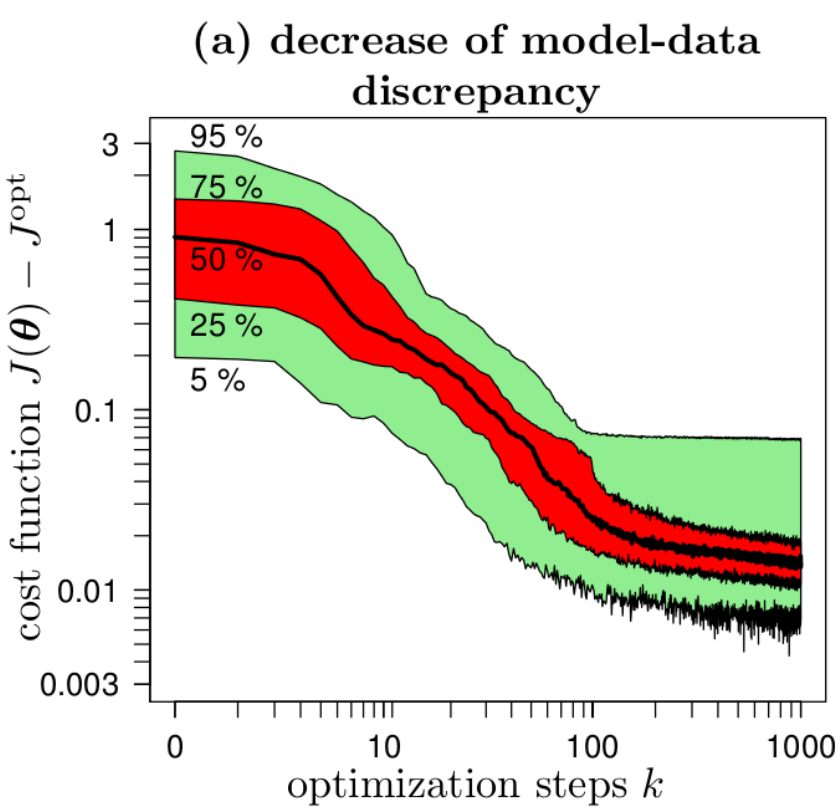
J Gradient estimation

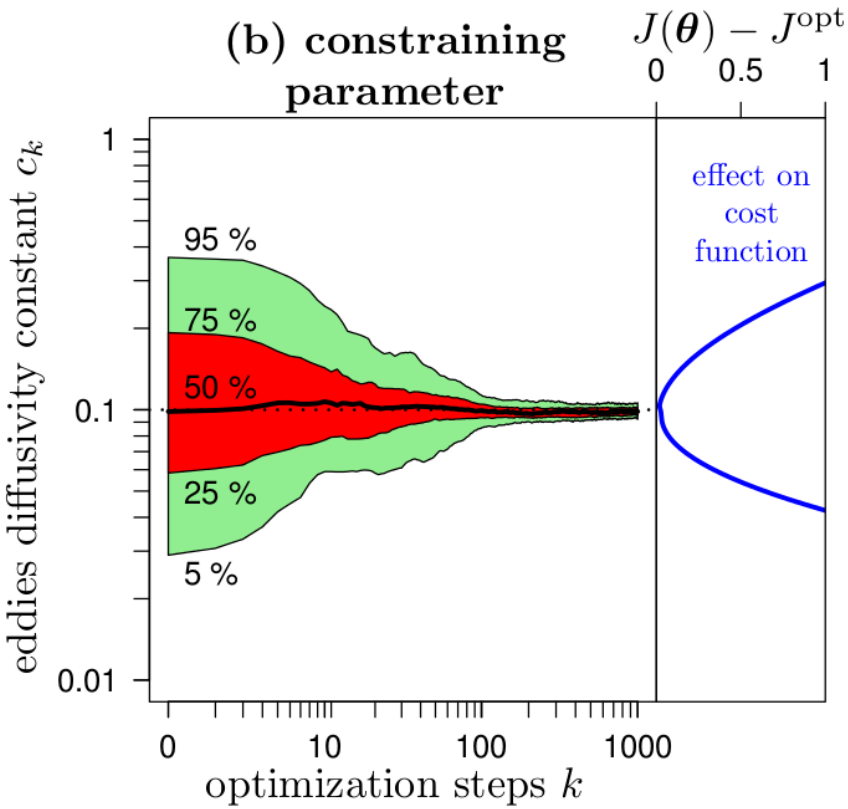
Classical : 1 Parameter at a time

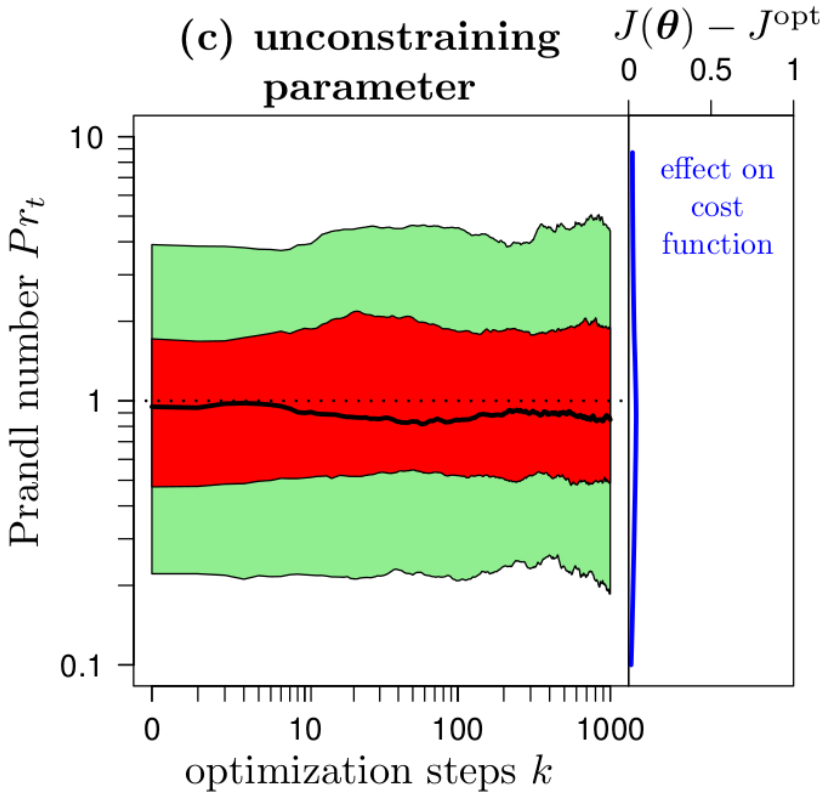
$$\nabla J(\boldsymbol{\theta^k}) = \begin{pmatrix} \frac{J(\boldsymbol{\theta^{(k)}} + \boldsymbol{\delta\theta_1^{(k)}}) - J(\boldsymbol{\theta^{(k)}} - \boldsymbol{\delta\theta_1^{(k)}})}{2c^{(k)}} \\ \vdots \\ \frac{J(\boldsymbol{\theta^{(k)}} + \boldsymbol{\delta\theta_i^{(k)}}) - J(\boldsymbol{\theta^{(k)}} - \boldsymbol{\delta\theta_i^{(k)}})}{2c^{(k)}} \\ \vdots \\ \frac{J(\boldsymbol{\theta^{(k)}} + \boldsymbol{\delta\theta_p^{(k)}}) - J(\boldsymbol{\theta^{(k)}} - \boldsymbol{\delta\theta_p^{(k)}})}{2c^{(k)}} \end{pmatrix}, \quad \boldsymbol{\delta\theta_i^{(k)}} = c^{(k)} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4}$$

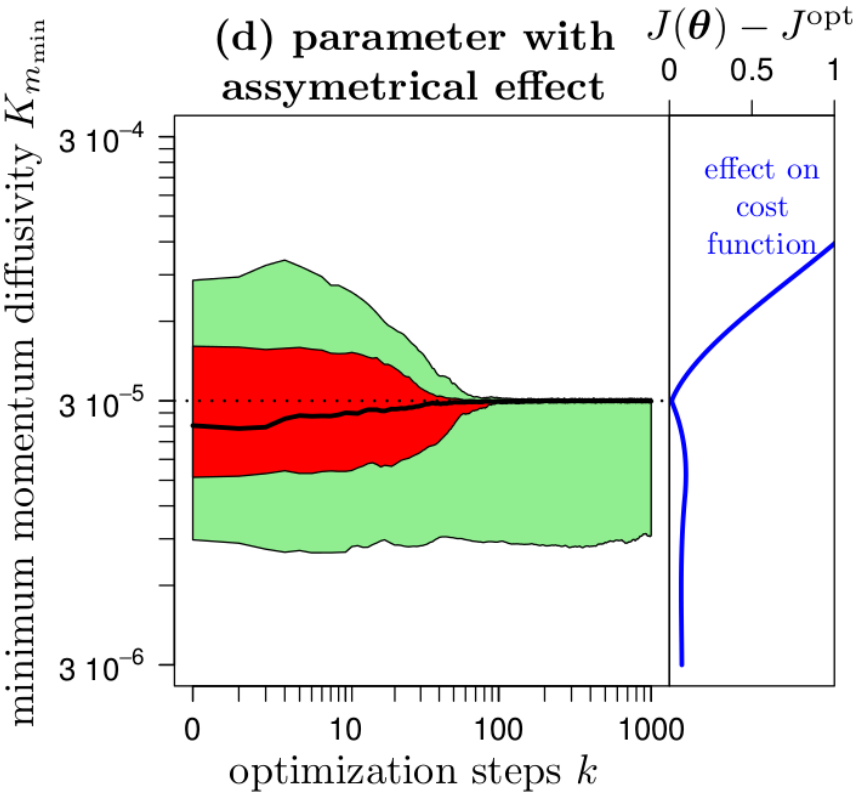
SPSA simultaneous Perturbations of All parameters

$$\nabla J(\boldsymbol{\theta^k}) = \begin{pmatrix} \frac{J(\boldsymbol{\theta^{(k)}} + \boldsymbol{\delta\theta^{(k)}}) - J(\boldsymbol{\theta^{(k)}} - \boldsymbol{\delta\theta^{(k)}})}{2c^{(k)}\Delta_1^{(k)}} \\ \vdots \\ \frac{J(\boldsymbol{\theta^{(k)}} + \boldsymbol{\delta\theta^{(k)}}) - J(\boldsymbol{\theta^{(k)}} - \boldsymbol{\delta\theta^{(k)}})}{2c^{(k)}\Delta_i^{(k)}} \\ \vdots \\ \frac{J(\boldsymbol{\theta^{(k)}} + \boldsymbol{\delta\theta^{(k)}}) - J(\boldsymbol{\theta^{(k)}} - \boldsymbol{\delta\theta^{(k)}})}{2c^{(k)}\Delta_p^{(k)}} \end{pmatrix}, \quad \boldsymbol{\delta\theta^{(k)}} = c^{(k)} \begin{pmatrix} \Delta_1^{(k)} \\ \vdots \\ \Delta_i^{(k)} \\ \vdots \\ \Delta_p^{(k)} \end{pmatrix} \tag{5}$$


(a) decrease of model-data discrepancy
cost function $J(\boldsymbol{\theta}) - J^{\text{opt}}$
3
1
0.1
0.01
0.003
95 %
75 %
50 %
25 %
5 %
0
10
100
1000
optimization steps k


(b) constraining parameter
$J(\boldsymbol{\theta}) - J^{\text{opt}}$
0
0.5
1
eddies diffusivity constant c_k
1
0.1
0.01
95 %
75 %
50 %
25 %
5 %
effect on cost function
0
10
100
1000
optimization steps k


(c) unconstraining parameter
$J(\boldsymbol{\theta}) - J^{\text{opt}}$
0
0.5
1
effect on cost function
Prandl number Pr_t
10
1
0.1
0
10
100
1000
optimization steps k


(d) parameter with assymetrical effect
$J(\boldsymbol{\theta}) - J^{\text{opt}}$
0
0.5
1
effect on cost function
minimum momentum diffusivity $K_{m_{\min}}$
$3\,10^{-4}$
$3\,10^{-5}$
$3\,10^{-6}$
0
10
100
1000
optimization steps k

Comments and perspectives

- Example of use of SPSA method for a turbulent closure scheme embeded in a one-dimensional vertical model of the ocean.

Parameters that constrain model predictions are estimated with a high accuracy.

- Some other parameters are well-estimated on average, but with less accuracy(The same as other methods)
- Results are obtained at a relatively low numerical cost.
- Allows to optimize models without an adjoint model costly to derive.
- Method fast enough to allow repetitions, to perform a sensitivity analysis
- Easy to change the optimization problem to solve (model, cost function, available data)
- Possibility to use for boundary conditiions fitting or in coupled hydrodynamic-biogeochemical models

Persistence extremal index method

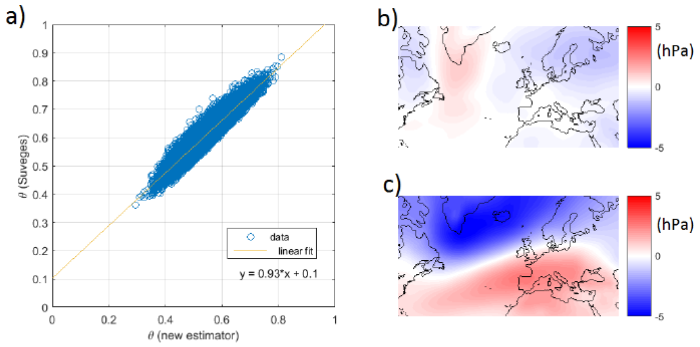
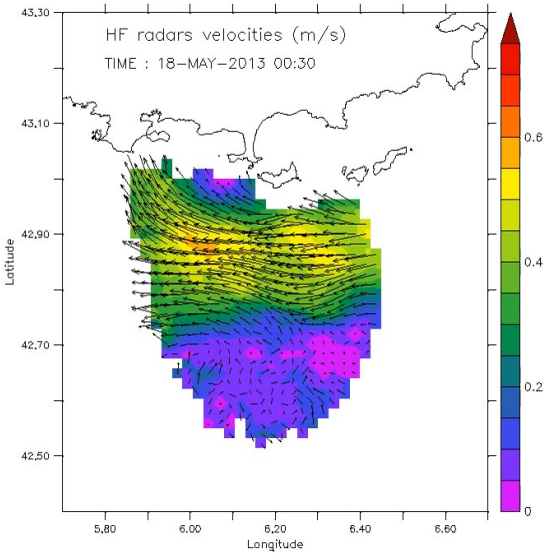


FIGURE 3. a) Scatter plot of $\hat{\theta}_{Su}$ (the Süveges estimator) vs $\hat{\theta}_5$ (the new estimator introduced in this work). Average map of the 5% sea-level pressure patterns such that the residual between $\hat{\theta}_{Su}$ and $\hat{\theta}_5$ are smaller than the 5% percentile (b) or larger (c) than the 95% percentile, for both taking as a threshold the 0.99-quantile of the observable distribution.

Caby, Faranda, Vaienti, Yiou, 2019



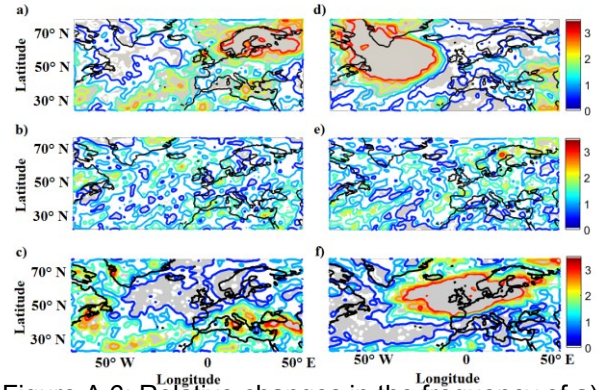
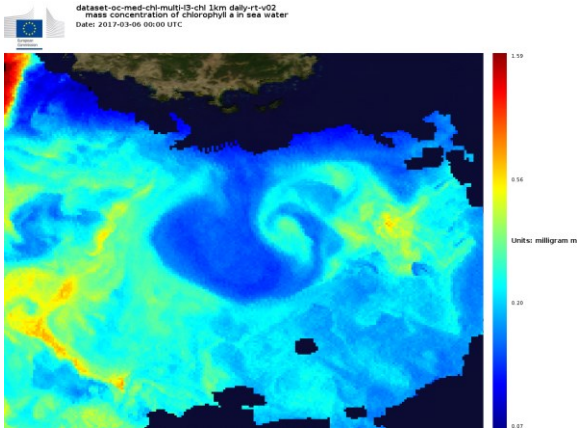


Figure A.6: Relative changes in the frequency of a), d) extreme cold; b), e) extreme wet and c), f) extreme 10m wind events for days with instantaneous dimension beyond the a-c) 0.98 and d-f) 0.02 quantiles of d in the ERA-Interim data. Contours start at 0, with an interval of 0.5; the grey shading shows statistically signicant changes. The maps in this gure are generated by MATLAB R2013a.

Faranda, Messori & Yiou 2019



Dumas, Granoullé & Guérin

Conclusions

- 3D observations of the marine surface layer in microtidal sea have been started (2DH HF radars + 1V ADP/CTD profilers)
- Sensitivity of the surface mixing layer to eddy viscosity as function of 3D space and time has been investigated (still in a very limited number of situations)
- A fast method for Identification of parameters of turbulent closure models (TKE, KPP, etc) has been proposed (tests in progress)
- A data analysis approach based on dynamical systems has been proposed for sparse data sets to classify events (from rare to extreme) and provide statistics (persistence)
- Ekman model needs to be revisited for high resolution modeling (surface and internal waves, Stokes drift, Langmuir circulations, horizontal variations)

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