Rock Flow Simulation by High-Order Quasi-Characteristics Scheme

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Introduction

- A pure second-order scheme of quasi-characteristics based on a pyramidal stencil is applied to the numerical modelling of non-stationary two-phase flows through porous media with the essentially heterogeneous properties.
- In contrast to well-known other high-resolution schemes with monotone properties, this scheme preserves a second-order approximation in regions, where discontinuities of solutions arise, as well as monotone properties of numerical solutions in those regions despite of well-known Godunov theorem.
- It is possible because the scheme under consideration is defined on a non-fixed stencil and is a combination of two high-order approximation scheme solutions with different dispersion properties.
- A special criterion according to which, one or another admissible solution is chosen, plays a key role in this scheme. A simple criterion with local character suitable for parallel computations is proposed.
- Some numerical results showing the efficiency of present approach in computations of two-phase flows through porous media with strongly discontinuous penetration coefficients are presented.

GOVERNING EQUATIONS

- Governing equations

$$m(\frac{\partial s}{\partial t}) - \frac{\partial}{\partial x}(\frac{kk_w}{\mu_w}\frac{\partial p}{\partial x}) - \frac{\partial}{\partial y}(\frac{kk_w}{\mu_w}\frac{\partial p}{\partial y}) = 0\,, \qquad (1)$$

$$\frac{\partial}{\partial x}[k(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o})\frac{\partial p}{\partial x}] + \frac{\partial}{\partial y}[k(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o})\frac{\partial p}{\partial y}] = 0\,. \qquad (2)$$

- Transformed transport equation

$$\frac{\partial s}{\partial t} - (\frac{k}{m\mu_w}\frac{\partial p}{\partial x}\frac{dk_w}{ds})\frac{\partial s}{\partial x} - (\frac{k}{m\mu_w}\frac{\partial p}{\partial y}\frac{dk_w}{ds})\frac{\partial s}{\partial y} = = \frac{k_w}{m}[\frac{\partial}{\partial x}(\frac{k}{\mu_w}\frac{\partial p}{\partial x}) + \frac{\partial}{\partial y}(\frac{k}{\mu_w}\frac{\partial p}{\partial y})]. \qquad (3)$$

INITIAL AND BOUNDARY CONDITIONS

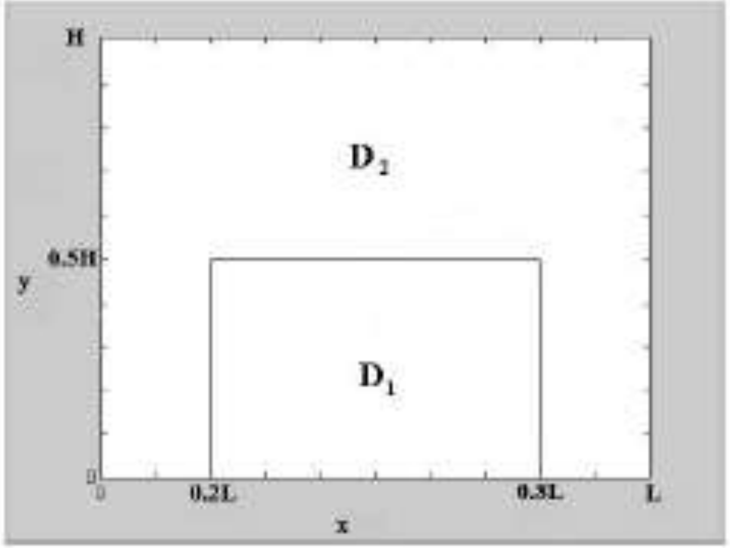


Fig.1. Flow region.

The absolute penetration factor k in each subregion is a constant function, therefore in all region we have

$$k = \begin{cases} k_{D_1}\,, & if \quad (x,y) \in D_1\,, \\ k_{D_2}\,, & if \quad (x,y) \in D_2\,. \end{cases} \tag{4}$$

Initial conditions for the transport equation (3) are

$$s(x,y,0) = \begin{cases} 0.2\,, & if \quad 0 \le x < L,\; 0 \le y \le H\,, \\ 1.0\,, & if \quad x = L,\; 0 \le y \le H \end{cases} \tag{5}$$

and the boundary conditions are

$$\begin{array}{c} \frac{\partial s}{\partial y} = 0\,,\; if \quad t > 0,\; y = 0, H,\quad 0 \le x \le L\,; \\ s(L,y,t) = 1.0\,,\; if \quad 0 \le y \le H\,,\quad t > 0\,. \end{array} \tag{6}$$

For the pressure equation (2) of the elliptic type we set a mixed Neumann and Dirichlet boundary conditions as follows

$$\begin{array}{c} \frac{\partial p}{\partial y} = 0\,,\; if \quad 0 < x < L\,,\; y = 0, H; \\ p = P_0\,,\; if \quad x = 0\,,\; 0 \le y \le H \\ \frac{\partial p}{\partial x} = \frac{Q_0 \mu_w}{H k k_w}\,,\; if \quad x = L,\; 0 \le y \le H\,. \end{array} \tag{7}$$

P_0 and Q_0 are known parameters here. The relative penetration factors of the water k_w and oil k_o are chosen as follows

$$k_w(s) = \begin{cases} 0\,, & if \quad s \le 0.1\,; \\ (\frac{s-0.1}{0.7})^3\,, & if \quad 0.1 < s \le 0.8\,; \\ 1\,, & if \quad s > 0.8\,; \end{cases} \tag{8}$$

$$k_o(s) = \begin{cases} 1\,, & if \quad s \le 0.1\,; \\ (\frac{0.8-s}{0.7})^3\,, & if \quad 0.1 < s \le 0.8\,; \\ 0\,, & if \quad s > 0.8\,. \end{cases} \tag{9}$$

Numerical Scheme

- Transport equation in generalized characteristic form

$$\frac{\partial u}{\partial t} + b_1 \frac{\partial u}{\partial x} + b_2 \frac{\partial u}{\partial y} = b_3,$$

- Transport equation in expanded characteristic form

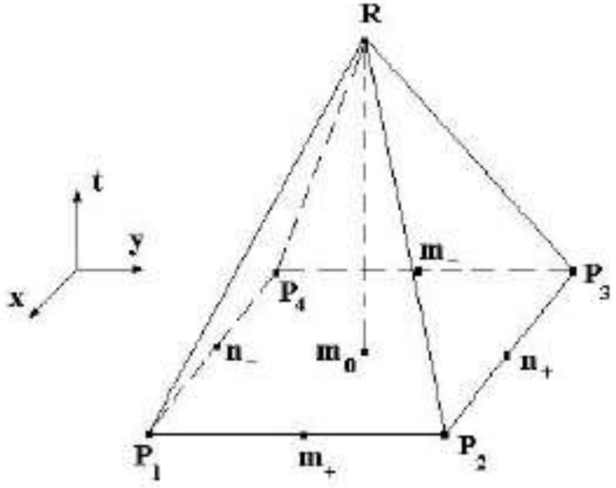
$$(\frac{du}{dt})_l + [b_1 - (\frac{dx}{dt})_l]\frac{\partial u}{\partial x} + [b_2 - (\frac{dy}{dt})_l]\frac{\partial u}{\partial y} = b_3$$

- Approximation of outward derivatives at the middle layer ($t_0 + \tau/2$)

$$(\frac{\partial u}{\partial x})_{t=t_0+\frac{\tau}{2}} = (\frac{\partial u}{\partial x})_C + (x - x_0)W + d(y - y_0)$$

$$(\frac{\partial u}{\partial y})_{t=t_0+\frac{\tau}{2}} = (\frac{\partial u}{\partial y})_C + (y - y_0)V + d(x - x_0)$$

Unknown variables: u_R, $(\frac{\partial u}{\partial x})_C$, $(\frac{\partial u}{\partial x})_C$, *d*



The pyramidal stencil.

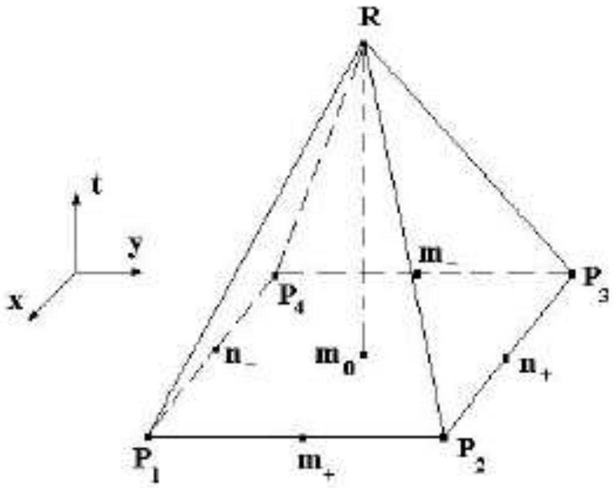
Scheme I

$$W_I = W(m_0) \equiv \frac{1}{3}[\frac{u_{P_1} - 2u_{n_-} + u_{P_4}}{h_x{}^2} + \frac{u_{m_+} - 2u_{m_0} + u_{m_-}}{h_x{}^2} + \frac{u_{P_2} - 2u_{n_+} + u_{P_3}}{h_x{}^2}],$$

$$V_I = V(m_0) \equiv \frac{1}{3}[\frac{u_{P_2} - 2u_{m_+} + u_{P_1}}{h_y{}^2} + \frac{u_{n_+} - 2u_{m_0} + u_{n_-}}{h_y{}^2} + \frac{u_{P_3} - 2u_{m_-} + u_{P_4}}{h_y{}^2}]$$

Here we take a center point of the middle section of our stencil $(t_0 + \frac{\tau}{2}, x_0, y_0)$ as the point C (or C_I) and choose values W and V (or W_I and V_I) at the middle layer $t_0 + \frac{\tau}{2}$

Numerical Scheme



The pyramidal stencil.

Scheme II

$$\left.\begin{array}{l} if\ \ b_1(x_0,y_0,t_0)\geq 0\ \ and\ \ b_2(x_0,y_0,t_0)\geq 0\ , \\ \quad then\ \ W_{II}=W(P_4),\ \ V_{II}=V(P_4)\ , \\ \qquad C_{II}=(P_{4x},P_{4y},t_0+\frac{\tau}{2})\ , \\ if\ \ b_1(x_0,y_0,t_0)\geq 0\ \ and\ \ b_2(x_0,y_0,t_0)<0\ , \\ \quad then\ \ W_{II}=W(P_3),\ \ V_{II}=V(P_3)\ , \\ \qquad C_{II}=(P_{3x},P_{3y},t_0+\frac{\tau}{2})\ , \\ if\ \ b_1(x_0,y_0,t_0)<0\ \ and\ \ b_2(x_0,y_0,t_0)<0\ , \\ \quad then\ \ W_{II}=W(P_2),\ \ V_{II}=V(P_2)\ , \\ \qquad C_{II}=(P_{2x},P_{2y},t_0+\frac{\tau}{2})\ , \\ if\ \ b_1(x_0,y_0,t_0)<0\ \ and\ \ b_2(x_0,y_0,t_0)\geq 0\ , \\ \quad then\ \ W_{II}=W(P_1),\ \ V_{II}=V(P_1)\ , \\ \qquad C_{II}=(P_{1x},P_{1y},t_0+\frac{\tau}{2})\ . \end{array}\right\}$$

Switching criterion – minimal principle

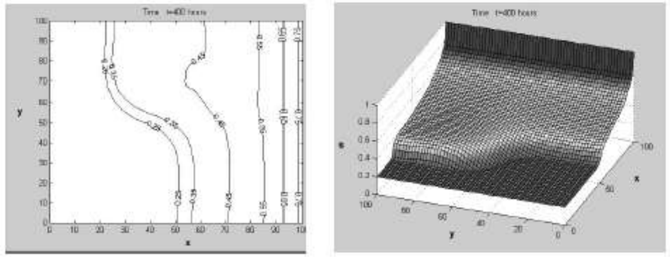
$$u_R{}^{final}=\min_{i=I,II}\left|u_R^i-\frac{C_0u_{m_0}+C_1u_{P_1}+C_2u_{P_2}+C_3u_{P_3}+C_4u_{P_4}}{C_0+C_1+C_2+C_3+C_4}\right|$$

In results of numerical tests –> the final form

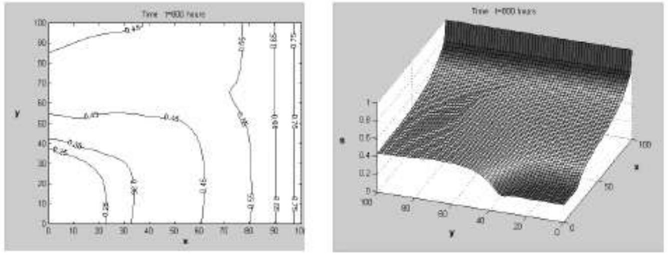
$$u_R{}^{final}=\min_{i=I,II}\left|u_R^i-u_{m_0}\right|$$

Computations

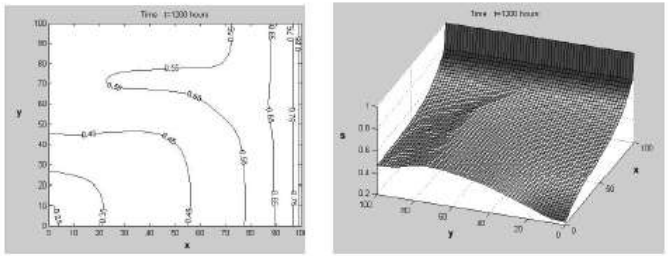
Now let us consider some numerical results obtained by the proposed method. Computations were carried out for the following values of parameters $m = 0.2$, $k_{D_2} = 1.0 \cdot 10^{-12}\ m^2$, $\mu_w = 1 \cdot 10^{-6}\ N \cdot sec \cdot m^{-2}$, $\mu_o = 3 \cdot 10^{-6}\ N \cdot sec \cdot m^{-2}$, $L = H = 100\ m$, $P_0 = 0$, $Q_0 = 0.69444 \cdot 10^{-12}\ m^2 \cdot sec^{-1}$. Parameter k_{D_1} varies in the range from $0.50 \cdot 10^{-12}\ m^2$ to $0.01 \cdot 10^{-12}\ m^2$. Thus the absolute penetration in the subregion D_1 is 2 to 100 times less than those in subregion D_2. Presented results correspond to the uniform grid with 61*61 nodal points in (x,y)-space.



The water saturation at $t = 400$ **hours.** $k_{D_1} = 0.50 \cdot 10^{-12}\ m^2$.

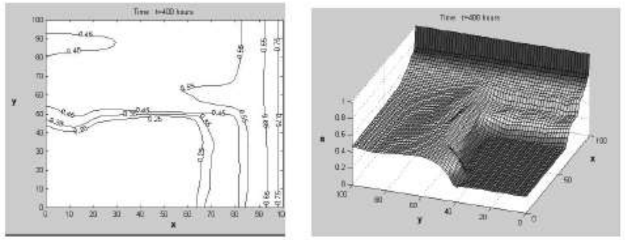


The water saturation at $t = 800$ **hours.** $k_{D_1} = 0.50 \cdot 10^{-12}\ m^2$.

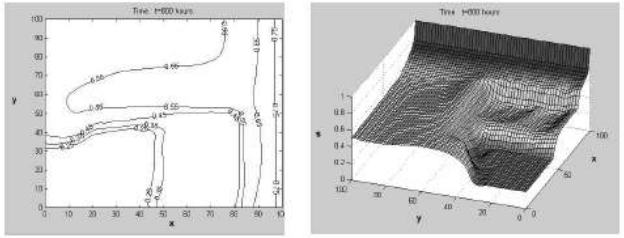


The water saturation at $t = 1200$ **hours.** $k_{D_1} = 0.50 \cdot 10^{-12}\ m^2$.

Computations



The water saturation at $t = 400$ **hours.**
$k_{D_1} = 0.20 \cdot 10^{-12}\ m^2$.



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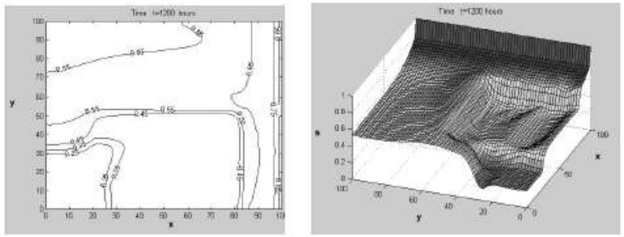

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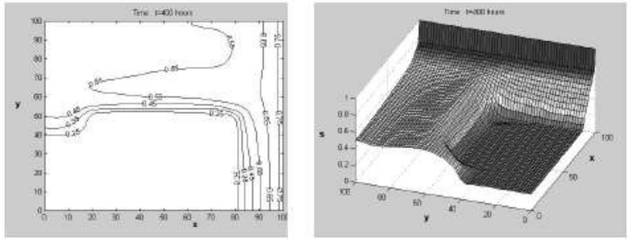
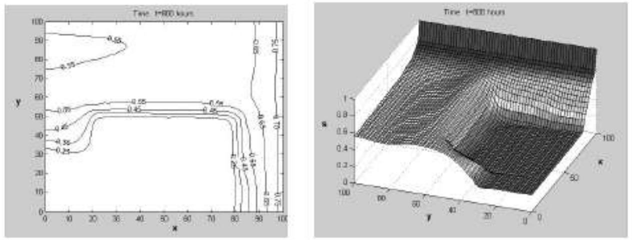
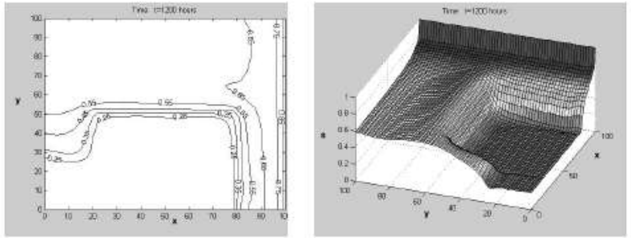


The water saturation at $t = 400$ **hours.**
$k_{D_1} = 0.01 \cdot 10^{-12}\ m^2$.



The water saturation at $t = 800$ **hours.**
$k_{D_1} = 0.01 \cdot 10^{-12}\ m^2$.



The water saturation at $t = 1200$ **hours.**
$k_{D_1} = 0.01 \cdot 10^{-12}\ m^2$.

Computations

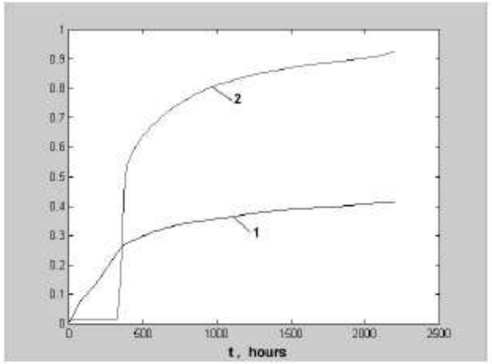
Characteristics of oil recovery efficiency

- The ratio of the recovery oil to the total oil volume

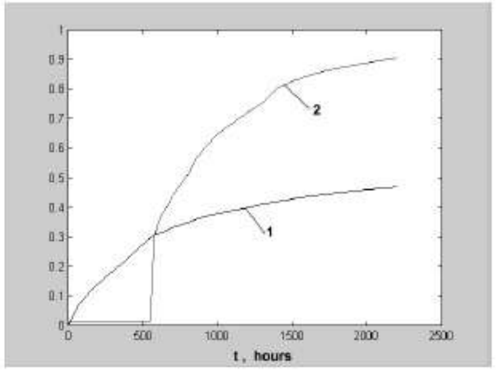
$$\theta(t) = \frac{\int_D [1 - s(x,y,t)]dxdy}{\int_D [1 - s(x,y,0)]dxdy}$$

- The water content in the development mixture at the production well

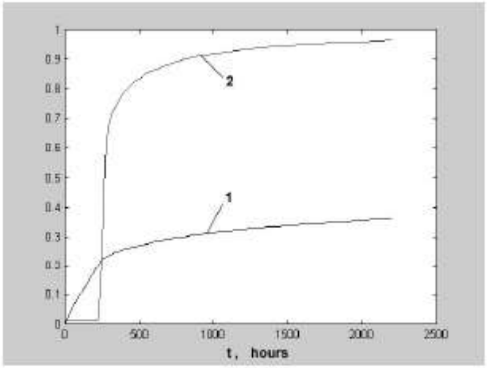
$$\gamma(t) = \frac{\int_0^L [k\frac{k_w}{\mu_w}\frac{\partial p}{\partial x}]_{x=0}dy}{\int_0^L [k(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o})\frac{\partial p}{\partial x}]_{x=0}dy}$$



Characteristics of efficiency of oil recovery. Line 1 - $\theta(t)$, line 2 - $\gamma(t)$. $k_{D_1} = 0.20 \cdot 10^{-12}\ m^2$.



Characteristics of efficiency of oil recovery. Line 1 - $\theta(t)$, line 2 - $\gamma(t)$. $k_{D_1} = 0.50 \cdot 10^{-12}\ m^2$.



Characteristics of efficiency of oil recovery. Line 1 - $\theta(t)$, line 2 - $\gamma(t)$. $k_{D_1} = 0.01 \cdot 10^{-12}\ m^2$.

CONCLUSIONS

- Our high-precision numerical quasi-characteristics technique developed for the transport equation allows us to obtain solutions of complicated porous media problem with essentially heterogeneous parameters without mesh fitting procedures on rough spatial meshes
- This technique can be implemented even on small computers and workstations for fast evaluation and exact modeling of oil and gas development technological processes

Thanks for attention!