# Numerical simulations of liquid drop dynamics in porous medium using adaptive mesh


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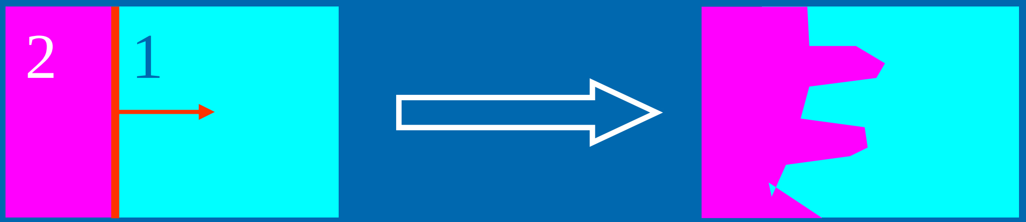
# Motivation

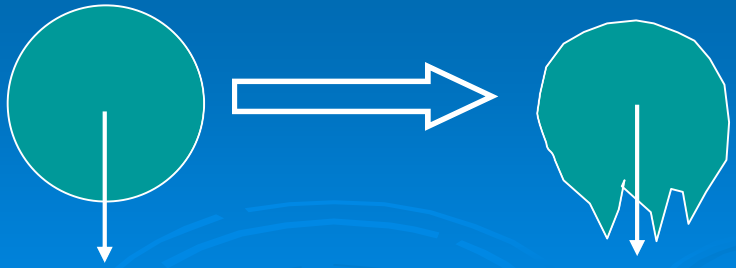
Darcy model

- Instability of a displacement front

$\eta_1 > \eta_2$




- Instability of a drop sedimentation



# Motivation

extraction of inclusions from a porous medium

Oil production: **compact inclusions** (c.i.) form up to 50% of oil deposits and could not be extracted

Different filtration processes

Analysis of behavior of c.i.

Active Control of c.i.

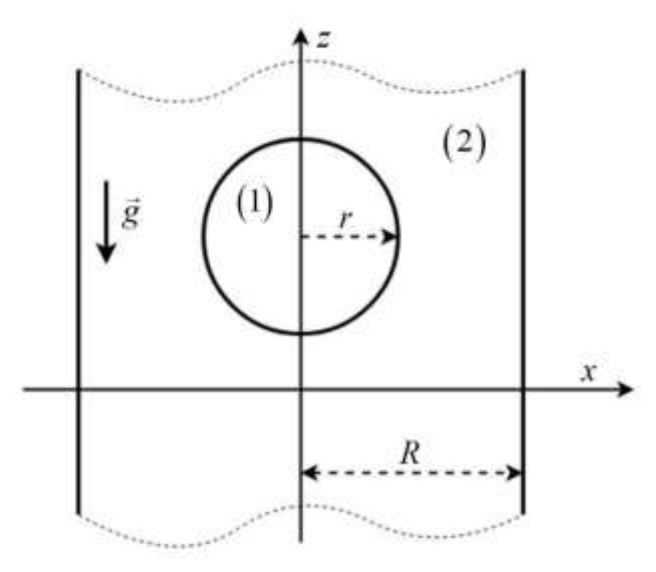
# Part 1. Numerical simulations drop sedimentation in porous medium

Darcy equation:

$$\nabla p + \frac{\eta}{k}\vec{u} = \rho g \vec{\gamma}$$

Governing equation in dimensionless form:

$$\nabla p + A\eta\vec{u} = \rho\vec{\gamma}, \quad \text{where} \quad A = \frac{\lambda - 1}{\mu + 0.5}$$




We use the following scales for variables:

$$[\rho] = \rho_1, \quad [\eta] = \eta_1, \quad [L] = R, \quad [u] = kg\frac{\rho_1 - \rho_2}{\eta_1 + 0.5\eta_2},$$

$$[t] = Kg\frac{\rho_1 - \rho_2}{\eta_1 + 0.5\eta_2}R, \quad [p] = \rho_1 gR.$$

# Numerical simulations of drop sedimentation in porous medium

Dimensionless parameters are

$$\lambda = \frac{\rho_2}{\rho_1},\ \mu = \frac{\eta_2}{\eta_1}$$ are dimensionless density and viscosity,

$$r = \frac{\tilde{r}}{R}$$ is drop radius

| | $\eta,\ g/cm\cdot s$ | $\rho,\ g/cm^3$ | $k,\ cm^2$ | $\varepsilon$ |
|---|---|---|---|---|
| Water (1) | 0,01 | 1,0 | $10^{-5}$ | 0,3 |
| Oil (2) | 0,015 | 0,7 | | |

Numerical algorithm:

- Level set method
- Adaptive mesh refinement
- Parallel computing

# Level set method

According the approach two-phase system is represented as one media which parameters sharply changes across the interface

Density and viscosity are calculated by distance function:

$$r(f) = l + (1 - l)H(f)$$

$$h(f) = m + (1 - m)H(f)$$

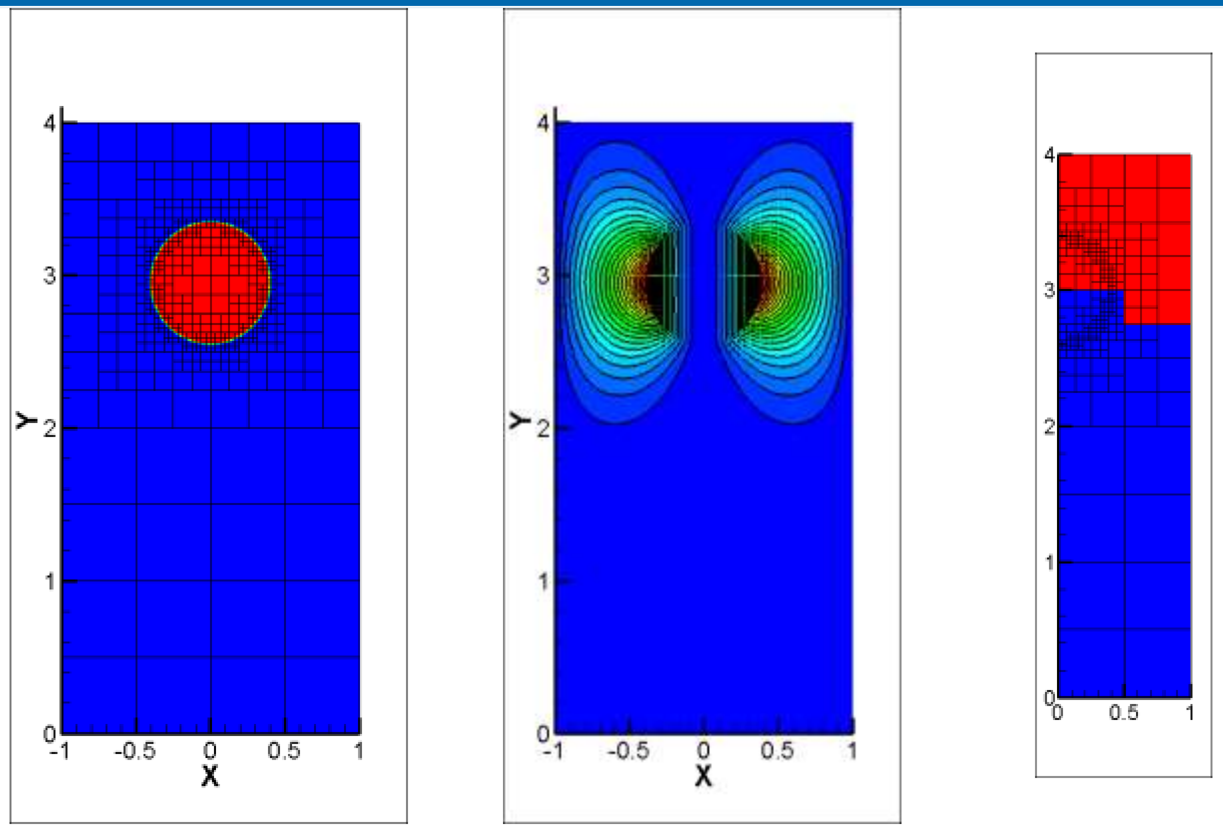
$$H_e(f) = \begin{cases} 0 & \text{if } f < -e \\ \frac{1}{2}\left[1 + \frac{f}{e} + \frac{1}{p}\sin(pf/e)\right] & \text{if } |f| \le e \\ 1 & \text{if } f > e \end{cases}$$

Calculations were perfumed for axisymmetric drop

$$\mathrm{v}_z = -\frac{1}{r}\frac{\partial \psi}{\partial r}, \quad \mathrm{v}_r = \frac{1}{r}\frac{\partial \psi}{\partial z}, \quad \Omega = \frac{\partial \mathrm{v}_z}{\partial r} - \frac{\partial \mathrm{v}_r}{\partial z}.$$

# Adaptive mesh refinement
# Parallel computing

*MacNeice P., Olson K.M., Mobarry C., Fainchtein R., Packer C. Paramesh: A parallel adaptive mesh refinement community toolkit // Computer Physics Communications. 2000. V.126. pp.330-354.*

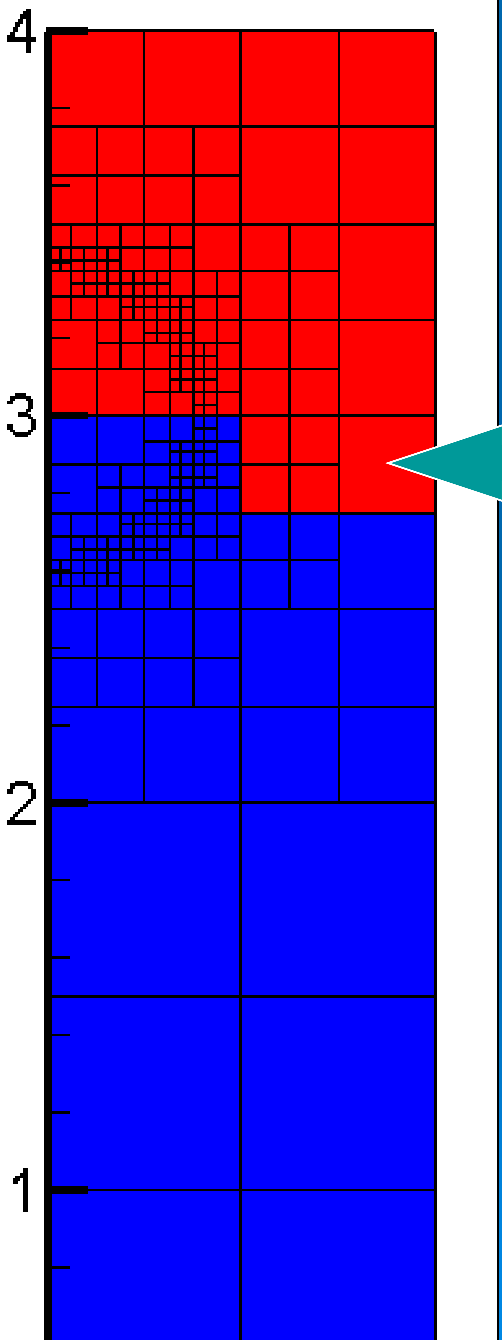


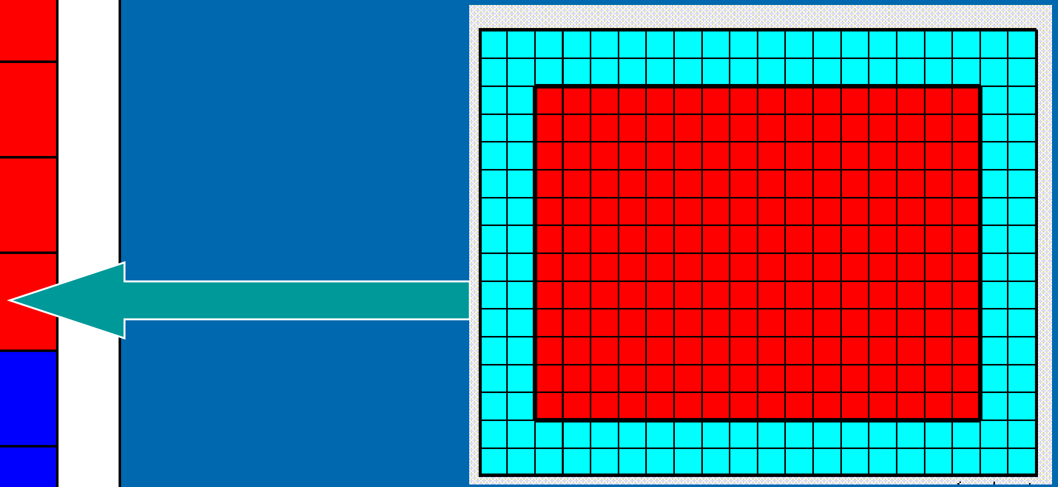

(a) (b) (c)

Axisymmetric drop sedimentation : (a) – initial drop shape and blocks of the mesh, (b) – flow function , (c) – distribution of mesh blocks among computation nodes

It is necessary to perform calculation with fine mesh near the interface

# Mesh sub-grids

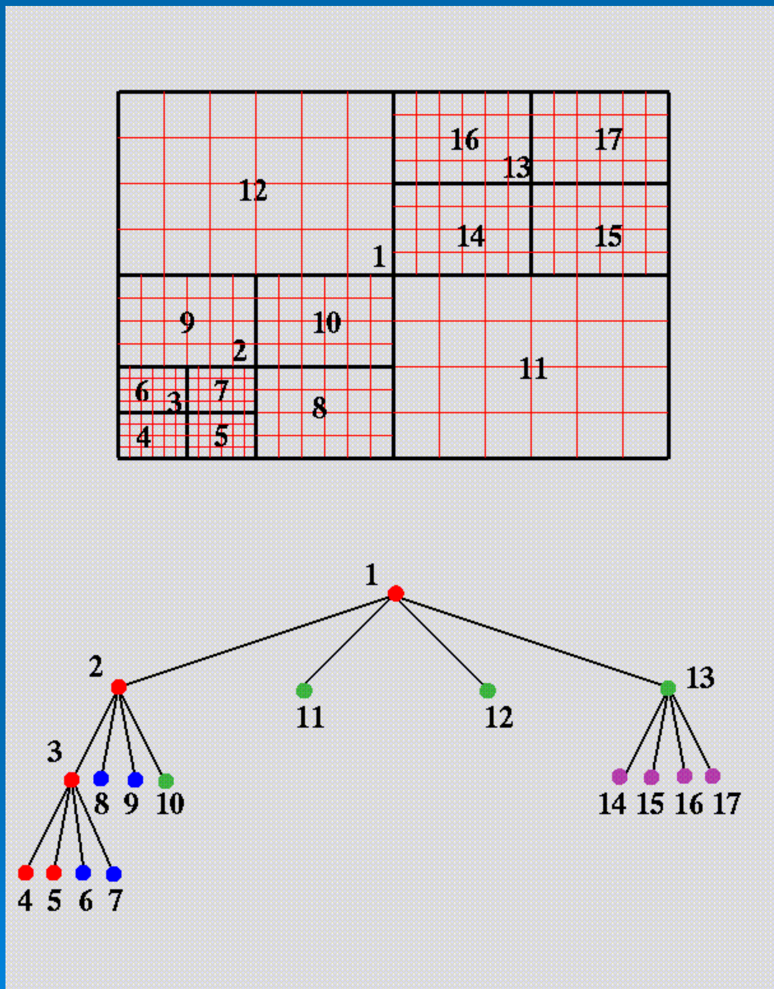


- The computational domain is covered with a hierarchy of numerical sub-grids.
- All the grid blocks have an identical logical structure. (ie the same number of grid points in each dimension, the same aspect ratios, the same number of guard cells, etc ). They are assumed to be logically cartesian (or structured).

. PARAMESH uses a block-structured adaptive mesh refinement scheme similar to others in the literatur

# Hierarchy of sub-grids

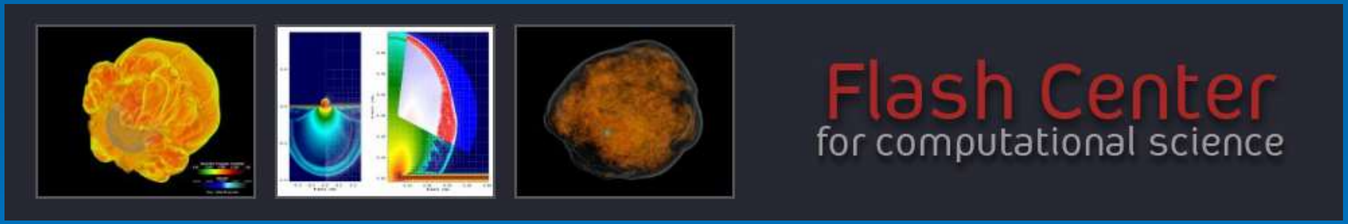



- The program creates a hierarchy of sub-grids to cover the computational domain, with spatial resolution varying to satisfy the demands of the application.
- These sub-grid blocks form the nodes of a tree data-structure.
- These sub-grids are distributed amongst the processors.
- PARAMESH uses a block-structured adaptive mesh refinement scheme In block-structured AMR, the fundamental data structure is a block of cells arranged in a logically Cartesian fashion. ``Logically Cartesian'' implies that each cell can be specified using a block identifier (processor number and local block number)

PARAMESH was written primarily by Peter MacNeice and Kevin Olson at NASA's Goddard Space Flight center as part of the NASA/ESTO-CT project (formally

https://sourceforge.net/projects/paramesh/

*MacNeice P., Olson K.M., Mobarry C., Fainchtein R., Packer C. Paramesh: A parallel adaptive mesh refinement community toolkit // Computer Physics Communications. 2000. V.126. pp.330-354.*




http://flash.uchicago.edu

The FLASH code is a publicly available high performance application code which has evolved into a modular, extensible software system from a collection of unconnected legacy codes.


http://www.cs.sandia.gov/CRF/aztec1.html

**Aztec** is a parallel iterative library for solving linear systems, which is both easy-to-use and efficient.

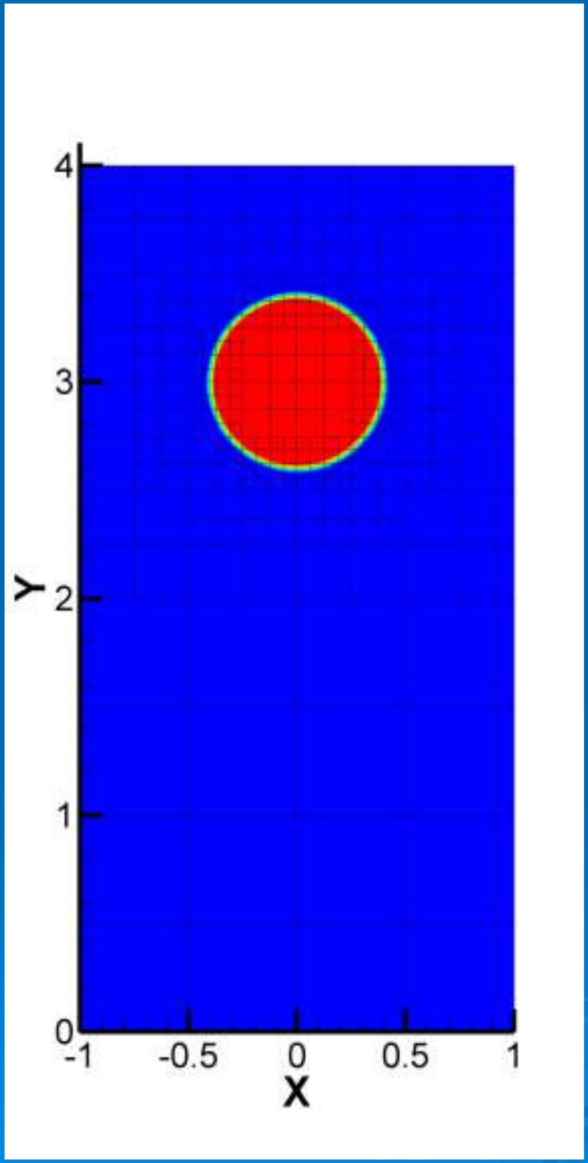
Trilinos

trilinos.org

The Trilinos Project is an effort to develop algorithms and enabling technologies within an object-oriented software framework for the solution of large-scale, complex multi-physics engineering and scientific problems.

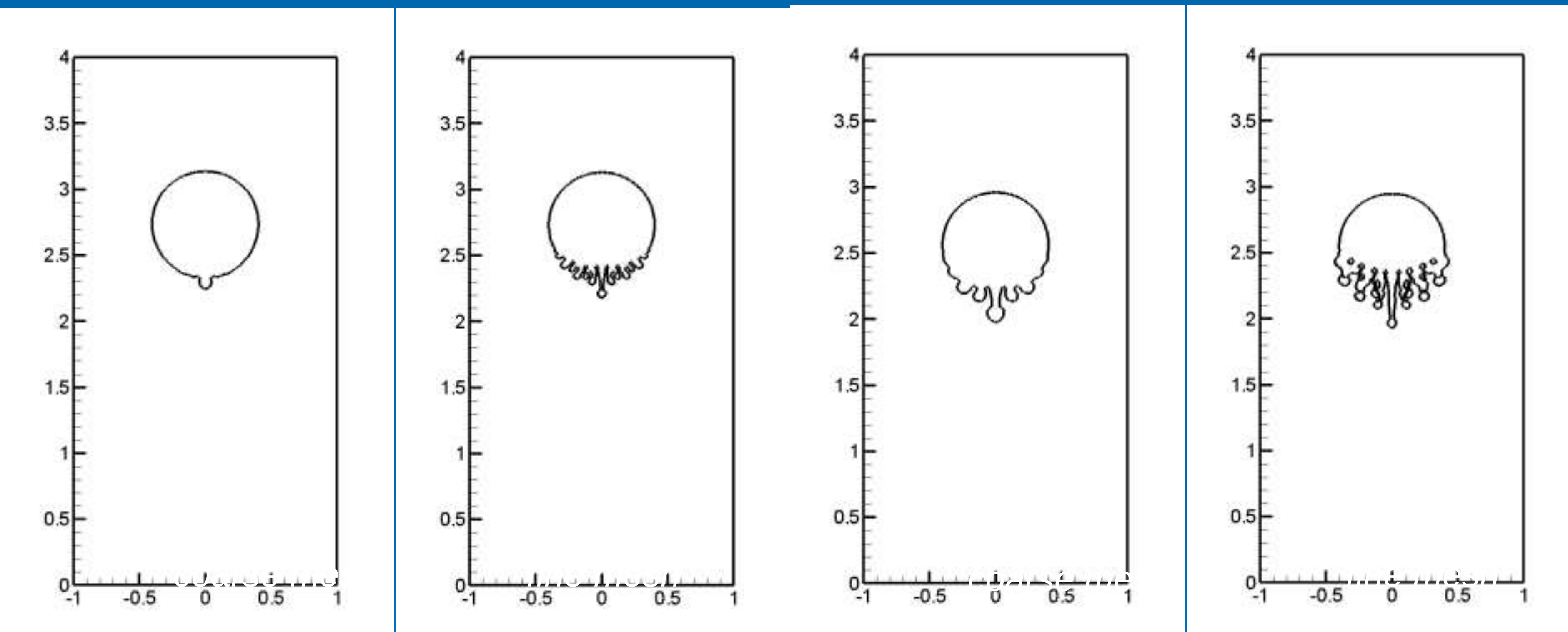
Trilinos 12.12 was released in September 2017

# Results of computations




Water drop sedimentation
in porous medium saturated by oil

# Numerical simulations drop sedimentation in porous medium

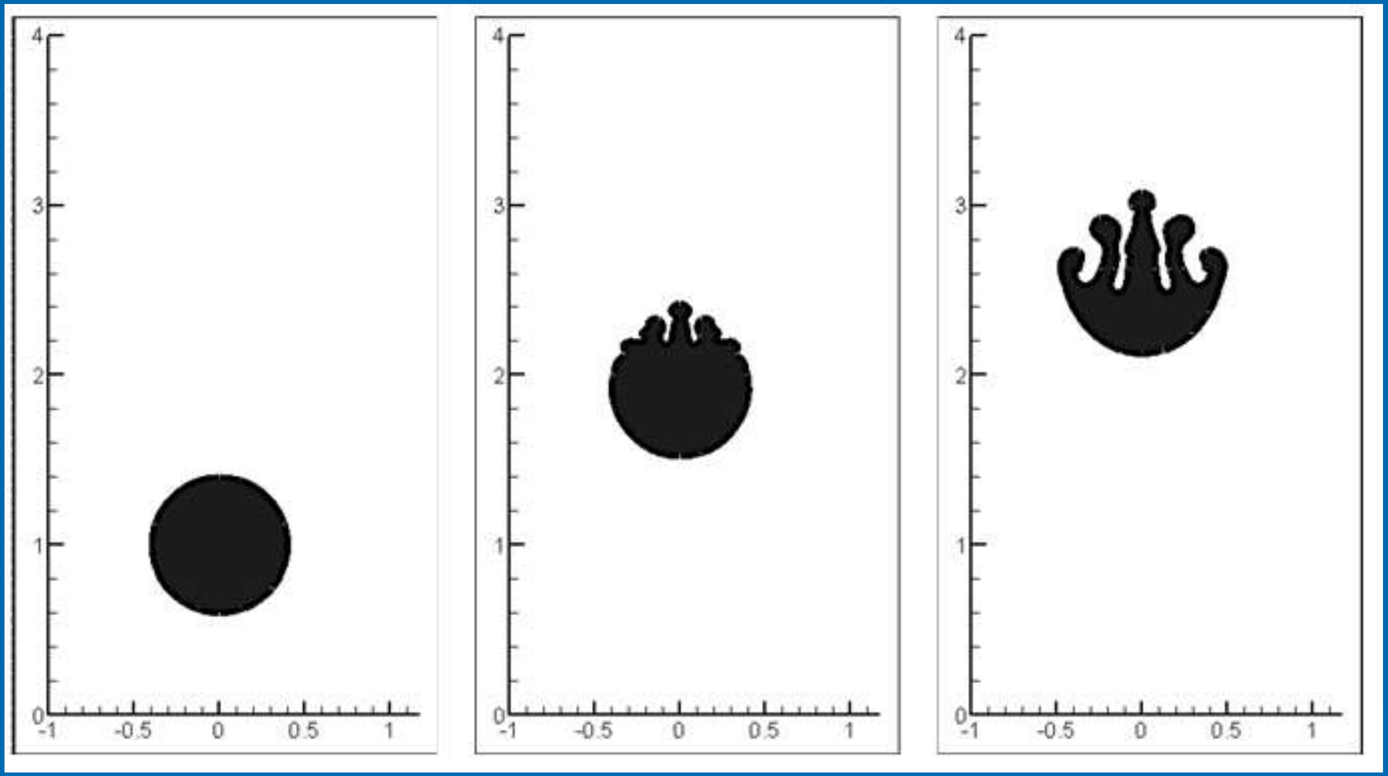



$t = 0.25$

$t = 0.5$

$$\lambda = 1.4 \quad \mu = 0.67 \quad r = 0.4$$

# Emersion of oil drop in water




$t = 0$ $t = 1$ $t = 1.5$

$\lambda = 1.4 \quad \mu = 0.67 \quad r = 0.4$

Inclusion is instable. Perturbations of interface always grow at the front of moving inclusion

# Part 2. Stability of inclusion under axial vibrations

Governing equations in dimensionless form:

$$\frac{\partial}{\partial t}(\rho\vec{u})+(\vec{u}\nabla)\rho\vec{u}=-\nabla p-R\eta\vec{u}+\left(-\frac{1}{\mathrm{Fr}}+a\cos t\right)\rho\vec{\gamma},$$

$$\mathrm{div}\vec{u}=0$$

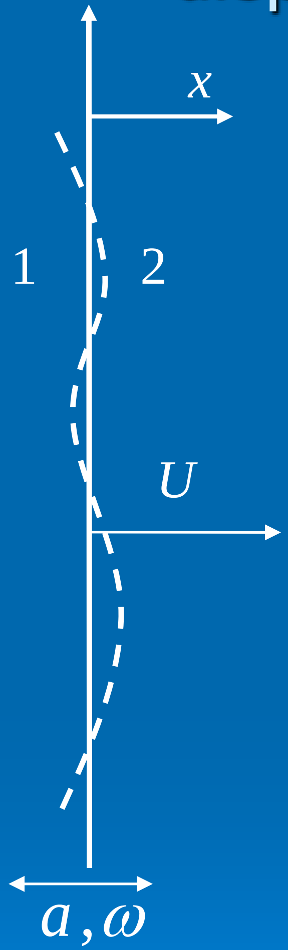
$$[\rho]=\rho_1 \quad [\eta]=\eta_1, \quad [L]=R, \quad [u]=\varepsilon R\omega, \quad [t]=\frac{1}{\omega}, \quad [p]=\rho_1 R^2\omega^2.$$

Dimensionless parameters:

$\mathrm{R}=\dfrac{\eta_1\varepsilon}{\rho_1 k\omega}$ is dissipation parameter, $\mathrm{Fr}=\dfrac{R\omega^2}{g}$ is Froude number

$a=\dfrac{\tilde{a}}{R}$ is vibration amplitude

# High-frequency vibration effect on the displacement front stability in porous medium



$\vec{\gamma} = \nabla x$

*D. V. Lyubimov and G. A. Sedel'nikov. Effect of Vibration on the Stability of a Plane Displacement Front in a Porous Medium. J. Fluid Dynamics, Vol. 41, No. 1, 2006, pp. 3–11.*

$$\lambda_0 = -\frac{U}{\varepsilon}\frac{\eta_1-\eta_2}{\eta_2+\eta_1}k - \frac{2a^2\omega^2 K(\rho_1+\rho_2)(\rho_1-\rho_2)^2}{\varepsilon(\eta_1+\eta_2)\left[(\rho_1+\rho_2)^2+\varepsilon^2(\eta_1+\eta_2)^2K^{-2}\omega^{-2}\right]}k^2$$

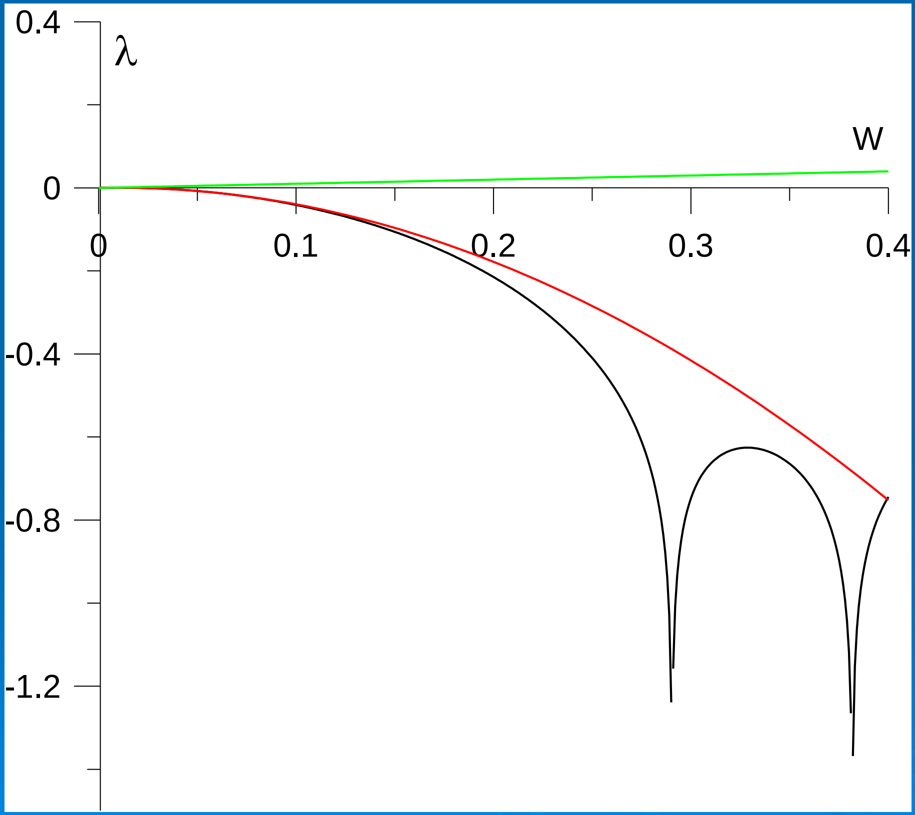
$$k_{cr} = \frac{U(\eta_2-\eta_1)\left[(\rho_1+\rho_2)^2+\varepsilon^2(\eta_1+\eta_2)^2K^{-2}\omega^{-2}\right]}{2a^2\omega^2K(\rho_1+\rho_2)(\rho_1-\rho_2)^2} \qquad k' = k_{cr}/2$$

$$T = \frac{2a^2\omega^2K\varepsilon(\eta_1+\eta_2)(\rho_1-\rho_2)^2}{U^2(\eta_1-\eta_2)^2(\rho_1+\rho_2)\left[1+\varepsilon^2(\eta_1+\eta_2)^2K^{-2}\omega^{-2}(\rho_1+\rho_2)^{-2}\right]}$$

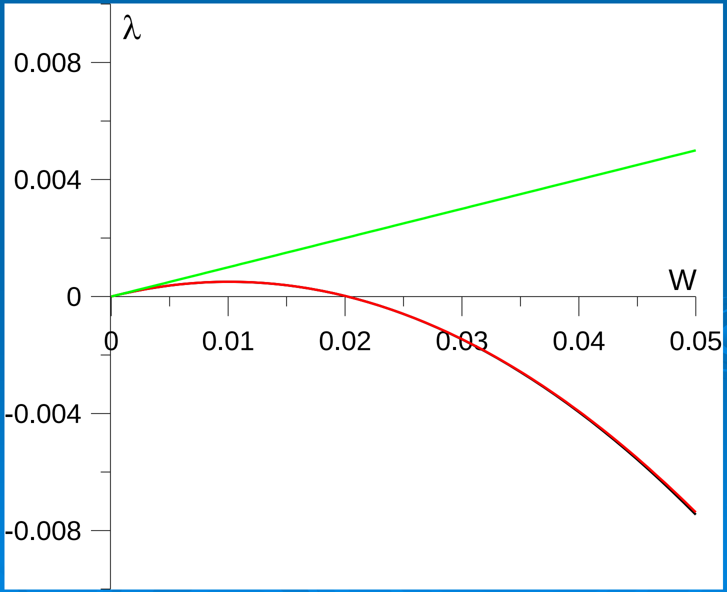
# Finite frequency vibrations effect on the displacement front stability in porous medium

$$\ddot{B} + \sigma \dot{B} - \mathrm{W}\left(1 + A\cos t\right)B = 0$$

$$\sigma = \frac{(\eta_1 + \eta_2)\varepsilon}{K(\rho_1 + \rho_2)\omega} = 0.7 \quad \mathrm{W} = \frac{U(\eta_2 - \eta_1)k}{K(\rho_1 + \rho_2)\omega^2} = 3.7 \cdot 10^{-5} \cdot k\,(sm^{-1}) \quad A = \frac{a\omega^2(\rho_1 - \rho_2)K}{U(\eta_1 - \eta_2)} = 947.5$$

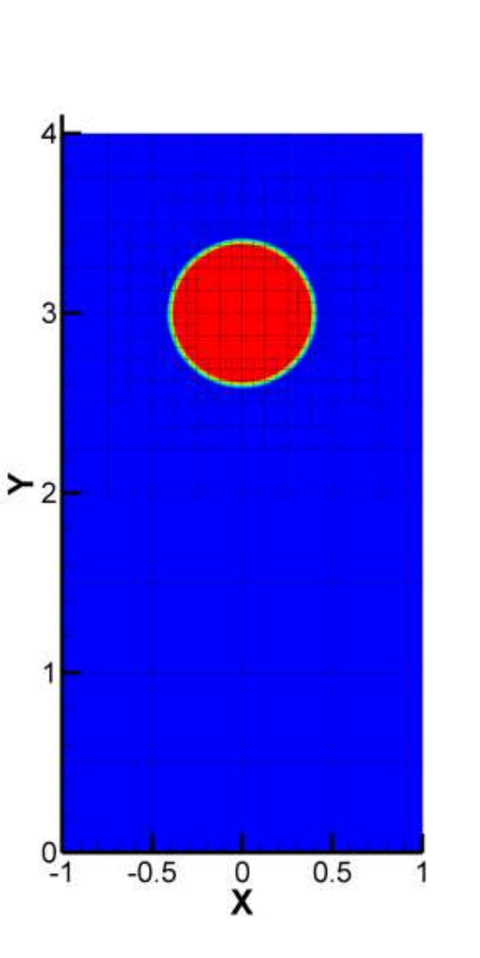



$\sigma = 10 \quad A = 1000$

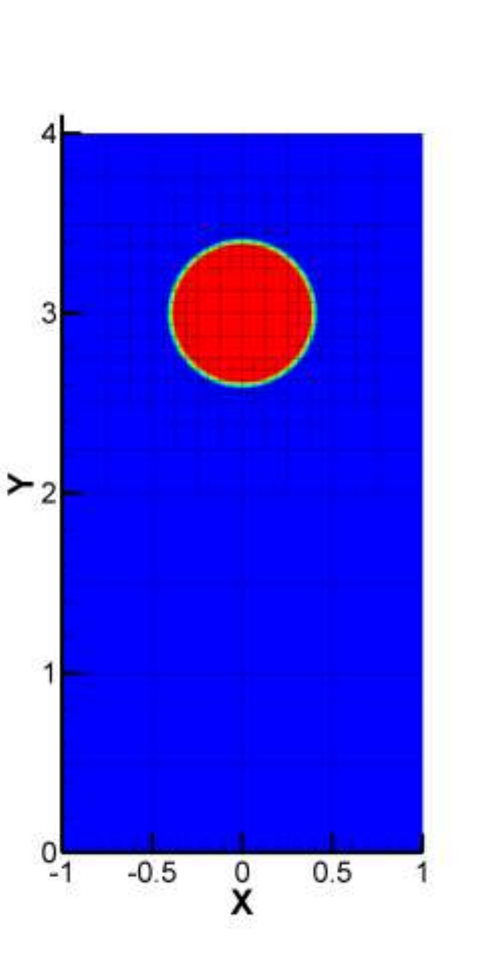


# Sedimentation of water drop in porous medium saturated by oil under axial vibrations

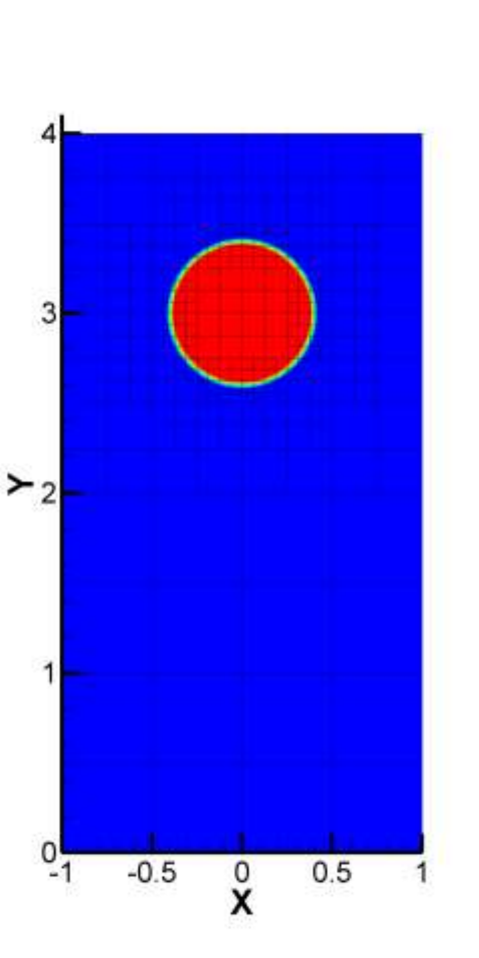
- ♦ Angular frequency eq. 100 1/c
- ♦ Chanel radius is 1 cm
- ♦ Drop radius is 0.4 cm




$a = 0.2$




$a = 0.5$

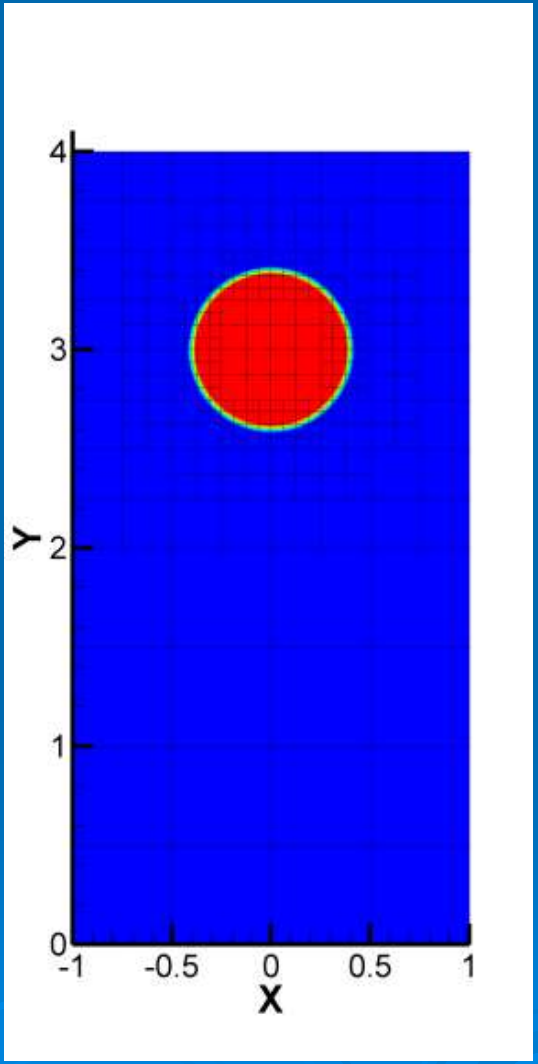



$a = 1$

# Kelvin-Helmholtz instability at high vibration intensity

$\omega = 100\ s^{-1}$

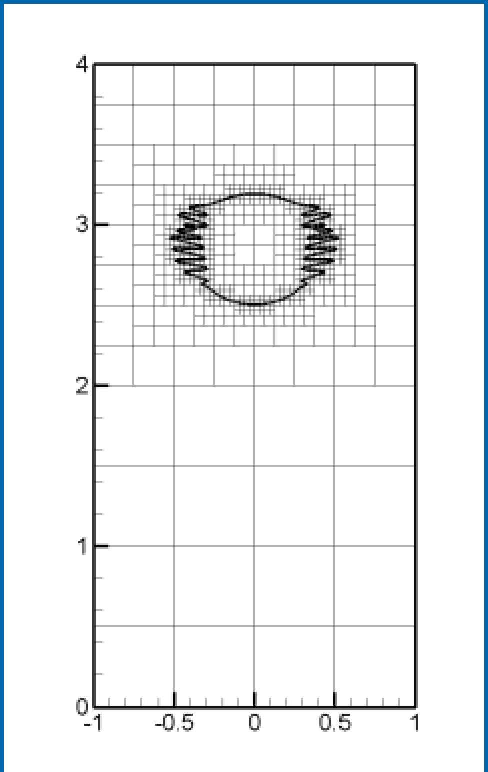
$a = 2\ cm$



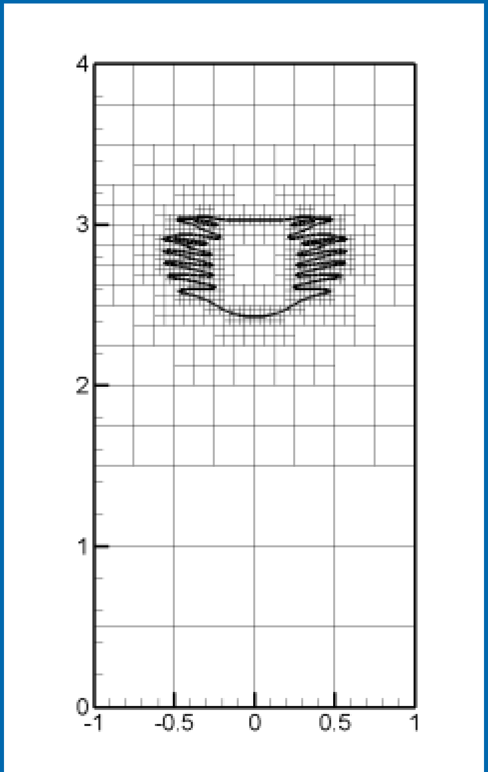
# Kelvin-Helmholtz instability at high vibration intensity

$\omega = 1000\ s^{-1}$

$a = 0.05\ cm$

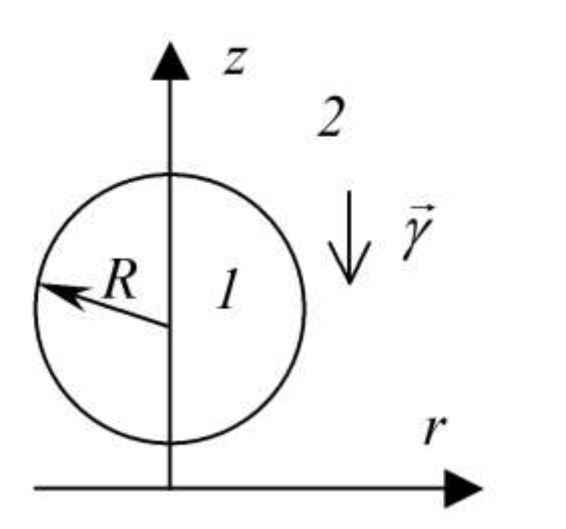



t = 0.3 c.




t = 0.65 c.

# Part 3. Influence of modulated pumping

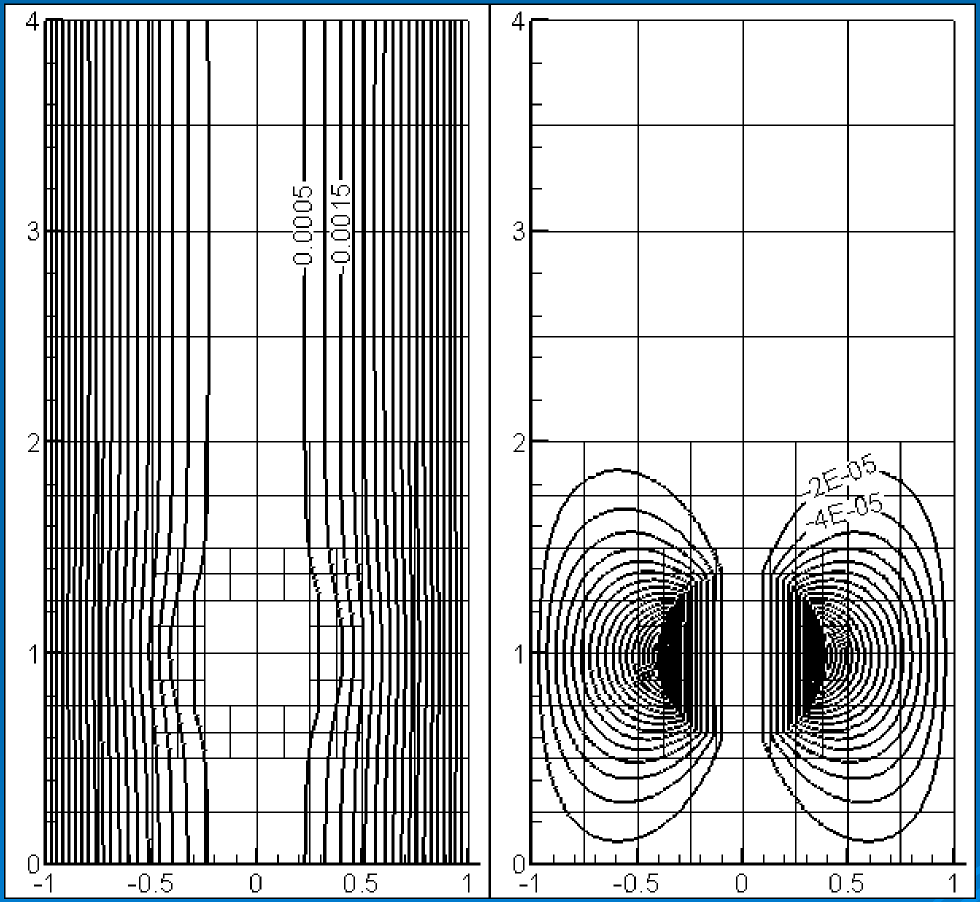



At upper and lower boundaries vertical component of velocity changes according the formula:

$$u_z = V\cos(\Omega t)$$

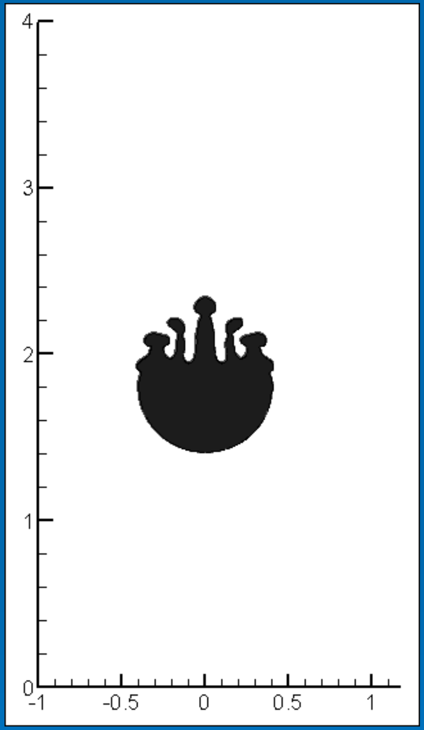
$V$ is dimensionless velocity amplitude
$\Omega$ is dimensionless frequency of external modulation.

# Flow function

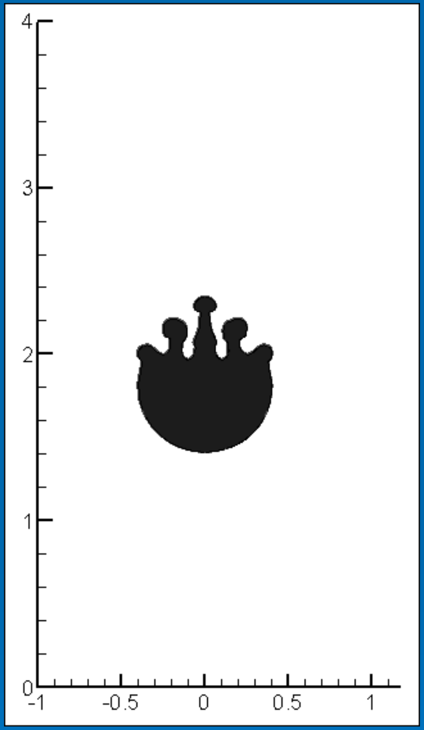



$V = 0.02$

# Results




a

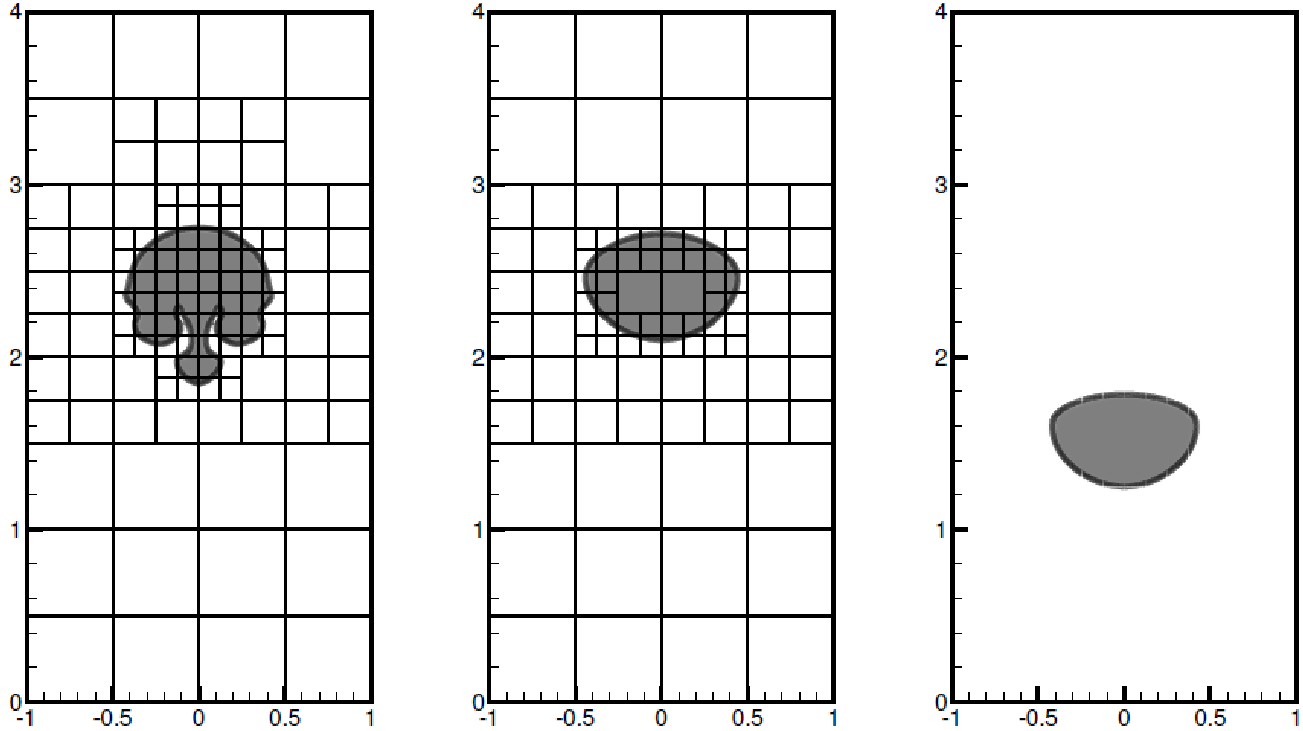



b

$V = 0.01,\ t = 15:$

a - $\Omega = 1$, b - $\Omega = 10$

# Drop stabilization under modulated pressure gradient



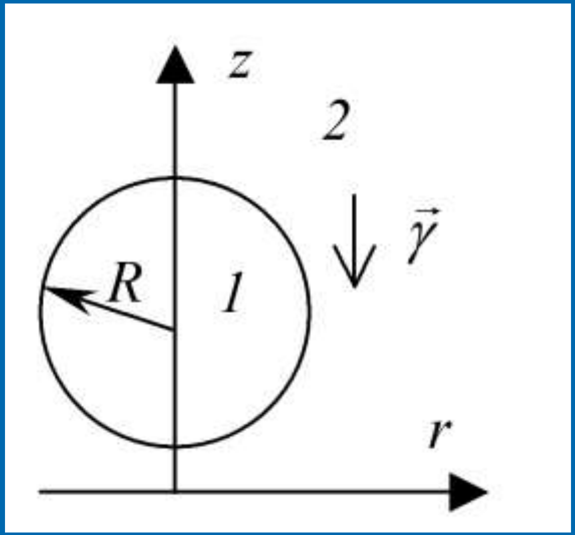

a) $t = 0.7$, $U = 10$

b) $t = 0.7$, $U = 20$

c) $t = 2$, $U = 20$

W = 1000

# Part 4. Simulation of droplet sedimentation using Buckley-Leverett model




$$v_1 + v_2 = 1$$

$$\frac{\partial v_1}{\partial t} + div\vec{\upsilon}_1 = 0$$

$$\vec{\upsilon}_1 = -\frac{K}{\eta_1} v_1 \left(\nabla P + g\rho_1\vec{\gamma}\right)$$

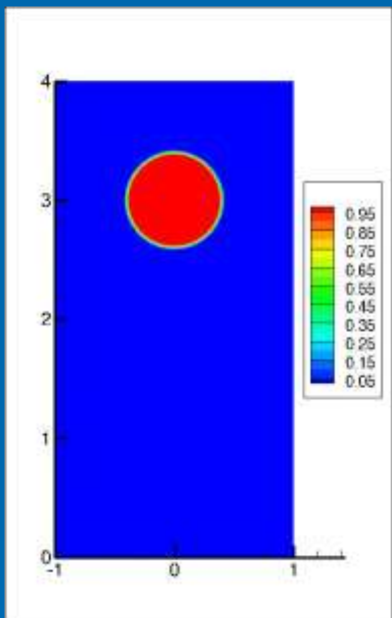
$$\frac{\partial v_2}{\partial t} + div\vec{\upsilon}_2 = 0$$

$$\vec{\upsilon}_2 = -\frac{K}{\eta_1} v_2 \left(\nabla P + g\rho_2\vec{\gamma}\right)$$

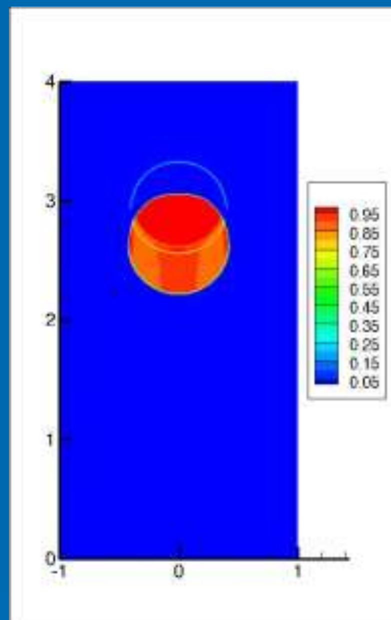
$$div\left[\frac{K}{\eta_1} v_1 \left(\nabla P + g\rho_1\vec{\gamma}\right) + \frac{K}{\eta_2} v_2 \left(\nabla P + g\rho_2\vec{\gamma}\right)\right] = 0$$

This equation for pressure is discretized by the finite volume method and is solved implicitly by the GMRES method with the preconditioning procedure using Aztec.

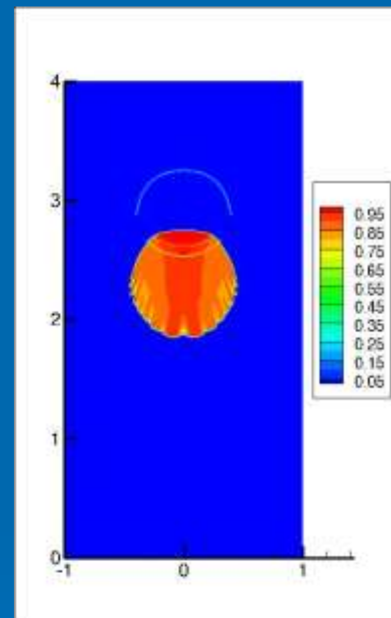
# Dynamics of a drop of more viscous fluid in porous medium saturated with less viscous fluid



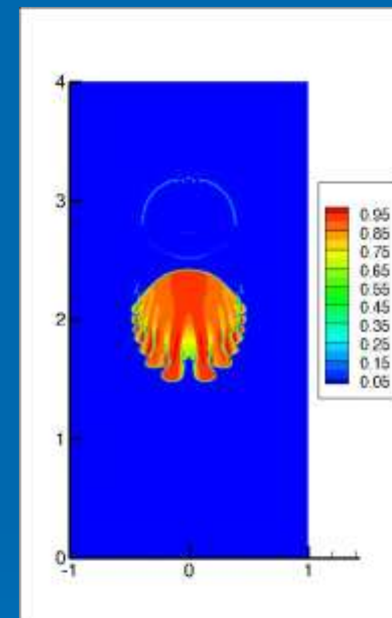

t = 0




t = 0.4



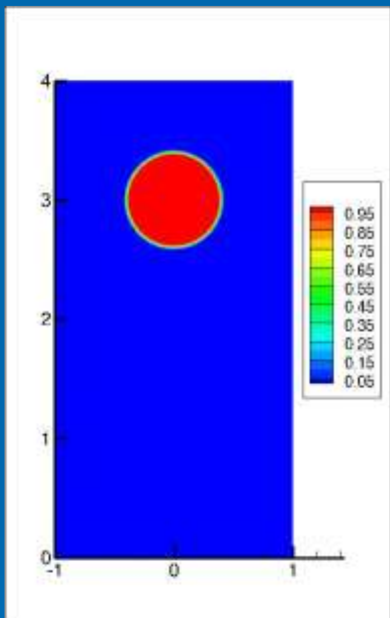

t = 1



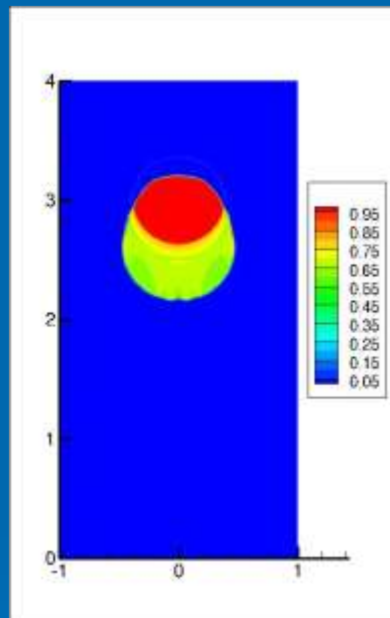

t = 1.5

$$\eta = 2, \quad \rho = 1.2$$

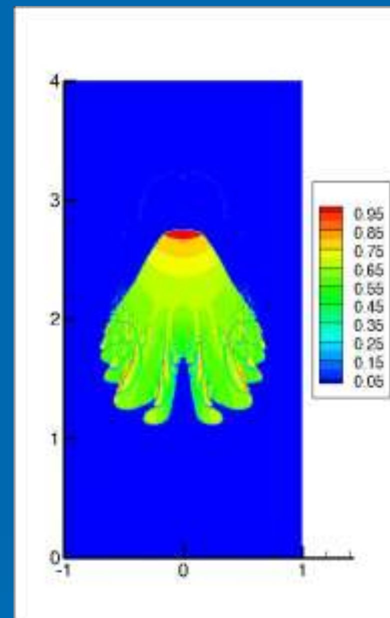
# Motion of a drop of less viscous fluid in porous medium saturated with more viscous fluid



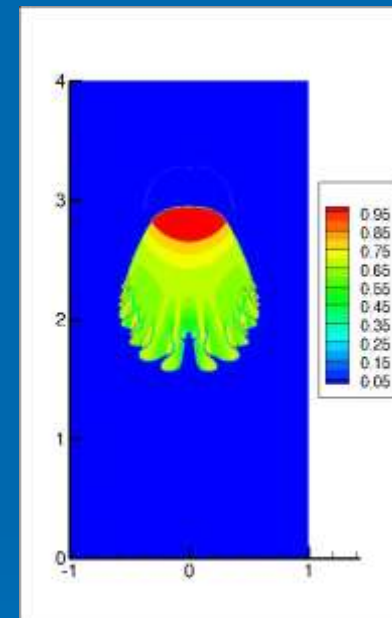

t = 0




t = 1



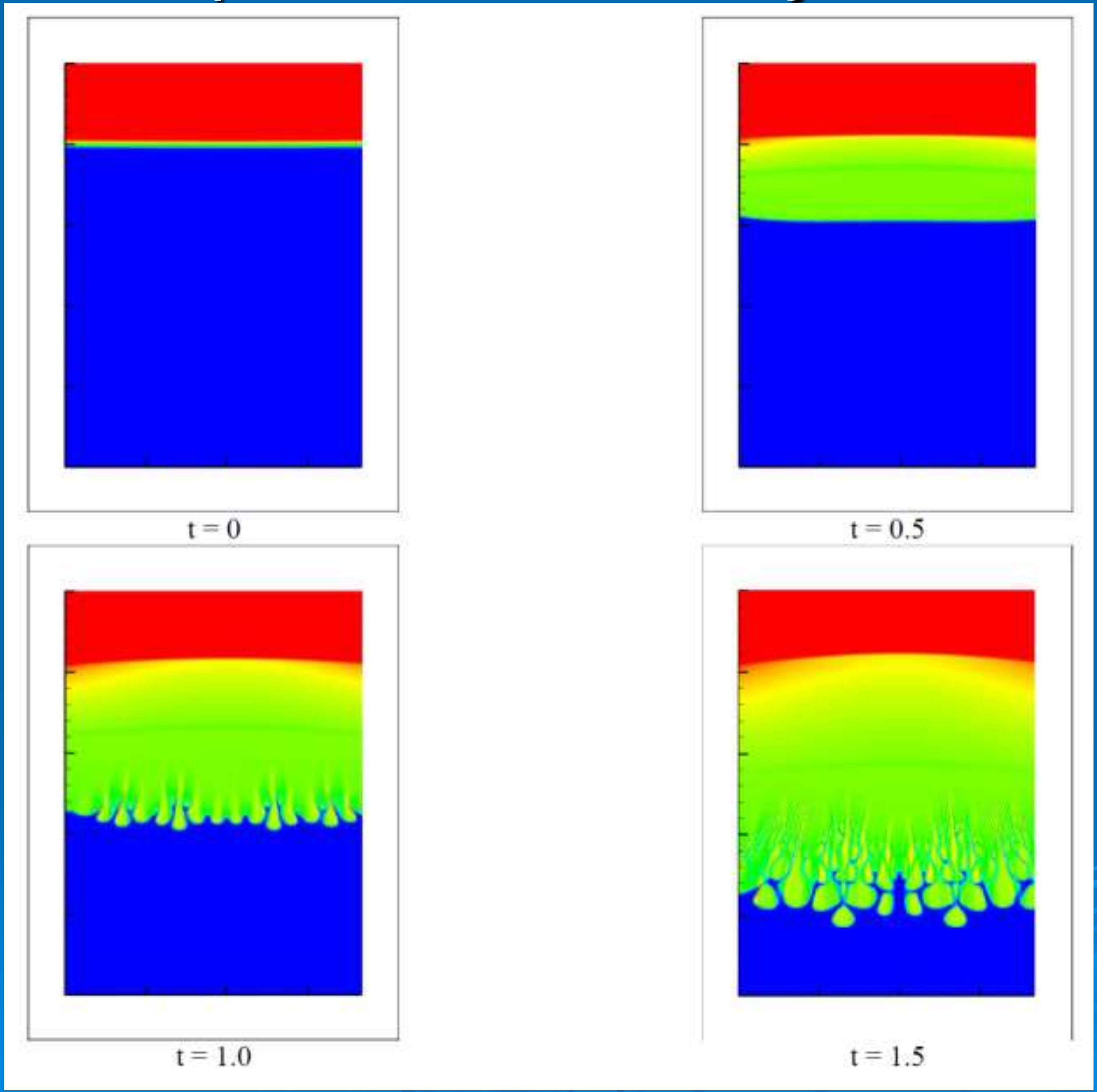

t = 2




t = 3

$$\eta = 0.5, \quad \rho = 1.2$$

# Displacement front dynamics




$$\eta = 0.5$$

$$\rho = 1.2$$

# Conclusions

Sedimentation or emersion of inclusion is instable. Perturbations of interface grow at the front of moving inclusion independently on viscosities values.

Vibrations can suppress short-wave perturbations of the displacement front, that are known to be most unstable (having the largest growth rate) in the classical non-vibrating case.

In the presence of weak vertical vibrations, similarly to the non-vibrating case, the droplet is unstable to small-scale perturbations localized near the front. Stronger vibrations can suppress the instability entirely.

Further increase of the strength of vibrations leads to another instability, this time localized at the droplet side.

Numerical simulation of the dynamics of a front in a porous medium, carried out within the framework of the Buckley-Leverett model, confirmed the presence of absolute instability of the displacement front in the case that the displacing fluid is less viscous. It is shown that the horizontal displacement front is stable at any ratio of viscosities in case the displacing fluid is less dense.

# Conclusions

If the density of the displacing fluid is greater than that displaced, then the horizontal displacement front is unstable, and the dynamics of the system depend on the ratio of the viscosities of the fluids.

In the case when the displacing fluid is more viscous, an instability develops similar to that previously observed in modeling the movement of a thin displacement front.

If the viscosity of the displacing fluid is less than that displaced, then the instability is associated with increase in the thickness of the displacement front. However, this secondary thin displacement front appear having smaller jump in the saturation of the media. The magnitude of the jump in saturation depends on the ratio of viscosities. The dynamics of the secondary front is similar to that observed in the case when the displacing fluid is more viscous.